Product Lines and Pricing for Aspirational Consumers*

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Abstract

I study pricing and product-line selection when consumers have a misplaced taste for variety over quality-differentiated products. Consumers make choices according to an aspiration-based procedure, and may purchase inferior or duplicative products. A multiproduct monopolist tries to divert demand to the highest-margin product and prunes low-margin products from its line. I juxtapose this model of a centralized seller with a model of single-product firm competition. Adding products can lower consumers' welfare by distracting them from better alternatives. However, competition still results in higher welfare than monopoly if the product line is fixed or there is no cost of returning products.

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1 Introduction

In foundational models of industrial organization, increased competition between firms provides a greater supply and variety of valuable products to consumers. This story seems in tension with the proliferation of low quality and duplicative product listings in some very competitive marketplaces. Online food delivery platforms have faced problems with 'virtual kitchens' where the same underlying menu is listed under many brand names (Wall Street Journal, 2023). Amazon and Spotify have taken steps to limit poor quality self-published books and audio content on their platforms (The Guardian, 2023; Forbes, 2025). Critics of fast fashion point to survey evidence suggesting short clothing lifespans (Mizrachi and Sharon, 2025) and quality deterioration (Cooper and Claxton, 2022).[1]

I study a model where consumer behaviour leads to demand for such products. Consumers make choices according to a procedure that results in a misapplied taste for variety over a set of products that are differentiated by quality but not taste. Under the choice procedure, consumers are only satiated by alternatives that are better than their average past consumption experience. Alternatives that have been chosen many times in the past end up being judged more harshly, resulting in the exploration of other options.

This choice procedure has features that make it interesting for studying product proliferation. In the long run, all products — even those of very low quality — are purchased with positive frequency. Moreover, adding duplicate or very similar products to the consumers' choice set cannibalizes demand for *all* products, even dissimilar ones. I consider the pricing and product-line selection decisions of a multiproduct monopolist selling to such consumers. I then contrast this with pricing and product availability when each product is controlled by a separate competitive firm. Despite the absence of taste-based variety in the set of products and the mistaken purchase of lower-quality items, competition continues to be beneficial for consumers in certain cases.

Consumers have unit demand and can choose from a set of vertically differentiated products, or choose to purchase none. A multiproduct monopolist tries to steer demand towards the product in its line that has the highest margin. It does this by setting the price of all other products equal to the level at which the consumer is indifferent between consuming the product and returning it if

[1] In Mizrachi and Sharon (2025), a survey of US consumers finds that 37.9% reporting disposing of clothing within one year of purchase, 14.2% within one month.

initially purchased. The monopolist prunes low-value products from its product line according to a cut-off rule.

Since consumers are not utility-maximizing, in principle it is ambiguous whether competition benefits consumers in terms of their true utility. I show that long-run consumer utility is higher under competition when the set of products placed on the market is fixed. However, owing to cannibalization effects, a multiproduct monopolist can have an incentive to prune certain products from the market that a single-product competitive firm does not. A monopolist may choose to remove from the market certain products that 'distract' consumers from choosing higher-utility alternatives. When consumers can return a product to the seller at no cost, competition still improves consumer welfare over monopoly — even with the distraction effect reduced due to the monopolist's pruning. Only when pruning is combined with products that are costly to return can a multiproduct monopolist deliver higher consumer welfare than competitive firms.

I call a consumer who makes choices according to the procedure an *aspirational consumer*. The aspirational consumer model is an adaptation of Gilboa and Schmeidler (1997)'s Cumulative Utility Consumer Theory, which is in turn a special case of the more general Case-Based Decision Theory introduced in Gilboa and Schmeidler (1995). A consumer chooses from a set M of products, with the outside option not to purchase any. Each product $k \in M$ has a true utility u_k that consumers derive from purchasing it. Utilities are fixed in an initial time period, and in each subsequent time period the consumer decides which alternative to choose. The consumer keeps track of their own past utilities and choices. Denote the fraction of times a consumer has chosen alternative k before time t by $\alpha_k(t)$. I assume aspirational consumers have an *aspiration level* given by $\sum_{j \in M} \alpha_j(t) u_j + \gamma$. This consists of the average utility realized by the consumer so far plus an additional *aspiration premium* $\gamma > 0$. For each product, consumers maintain an index composed of the number of times that alternative has been chosen multiplied by the difference between the utility of that alternative and the aspiration level. In each period the consumer chooses the alternative that has the highest index.

Aspirational consumers' choice procedure uses past consumption experiences relative to an average quality baseline to rank alternatives. If the typical consumption experience was at the standard of a Michelin-star restaurant, then a restaurant would have to meet a high bar to be counted positively in that prod-

uct's index. The cumulation of the indices by choice frequency and the aspiration premium $\gamma > 0$ give aspirational consumers a 'taste for variety' even though all consumers agree on the quality ranking over products. As an alternative is chosen more frequently, the aspiration level converges towards the utility of that product plus the aspiration premium γ. The aspiration premium reflects that the consumer wants a product's utility to exceed the average they have obtained so far by some margin. As the aspiration level converges, eventually the index for the frequently chosen product will drop below the indices of other products. The procedure thus captures a satiation effect which causes consumers to switch between different products over time.

This switching away from preferred options is consistent with some evidence from psychology. There is work suggesting loss of pleasure due to variety seeking (Ratner et al., 1999), underestimation of the level of repeated consumption required before satiation (Read and Loewenstein, 1995) and the potential manipulation of satiation based on recall of past consumption (Galak et al., 2009; Galak and Redden, 2018). In line with this, Puig (2024) presents an axiomatization of sequential choice exhibiting a taste for variety that differs from the standard expected utility axiomatization.

I prove that in the long run, aspirational consumers' choice frequencies converge to steady-state levels. Denote the long-run aspiration level by $\overline{u}^*$. This aspiration level will exceed the utility of any individual alternative: for all k, $\overline{u}^* > u_k$. The long-run frequency α_k^* with which any alternative k is chosen must satisfy the following relationship

$$\alpha_k^*\left(\overline{u}^* - u_k\right) = \frac{\gamma}{m+1} \tag{1}$$

where $m+1$ is the number of available alternatives. Since $\gamma > 0$ and $\overline{u}^* - u_k > 0$, all products are consumed with some frequency in the long run: $\alpha_k^* > 0$. The following example demonstrates that choice frequencies that satisfy this condition are subject to a 'duplicates property', and that the long-run aspiration level can fall when duplicates are added.

Example 1. An aspirational consumer has to decide what type of coffee to drink, with the option to abstain from consuming. The true utility of an espresso is 35, while drinking instant coffee gives zero utility, the same as not drinking at all. Table 1 outlines long-run choice frequencies that satisfy the condition (1) for all alternatives. We consider the choice frequencies when only one brand of instant

coffee is available, and contrast with choice frequencies when three duplicative brands are added.

Table 1: Long-run frequency of beverage choice

	$m_1 = 2,\ \frac{\gamma}{m_1+1} = 12$			$m_2 = 5,\ \frac{\gamma}{m_2+1} = 6$		
	$\overline{u}^* - u_k$	α_k^*	$\alpha_k^* \cdot [\overline{u}^* - u_k]$	$\overline{u}^* - u_k$	α_k^*	$\alpha_k^* \cdot [\overline{u}^* - u_k]$
Espresso (35)	21	4/7	12	15	2/5	6
No Coffee (0)	56	3/14	12	50	3/25	6
Instant (0)	56	3/14	12	50	3/25	6
Instant copy (0) ×3	—	—	—	50	3/25	6
Aspiration level $\overline{u}^*$		56			50	

We see that adding the clone instant coffee brands reduces the frequency with which the espresso coffee and no-purchase alternatives are chosen, not just the frequency with which the original instant coffee brand is chosen. △

If we subtract the aspiration premium γ from the aspiration level, we obtain the long-run average utility of the consumer. Example 1 therefore demonstrates how adding products to the choice set can reduce long-run consumer welfare. Since a single-product firm can always generate some demand for its product at a profitable price, it will always put its product on the market if possible. Table 1 shows how this could potentially reduce consumer welfare. However, I demonstrate that if we account for the equilibrium behaviour of single-product firms that compete in prices, then in many cases competition still improves long-run consumer welfare relative to a multiproduct monopolist. This is true if the multiproduct monopolist cannot prune the set of products it puts on the market, or when consumers can return a product to the seller at zero cost. Thus competition can still be a powerful force for increasing consumer welfare even with these boundedly rational consumers.

The aspirational consumer model provides an extremely tractable stochastic demand system with which to analyse these questions. In Section 5, I discuss how it relates to other models of stochastic choice, as well as existing work in industrial organization and the spread of duplicate products in marketplaces. Aspirational consumers' long-run choice frequencies do not satisfy the Independence of Irrelevant Alternatives (IIA) property of Luce (1959). In addition, they violate the regularity property required for consistency with a random utility model.

Unlike in many models of stochastic choice, randomness emerges from consumers' bounded rationality rather than shocks to preferences. This allows a clear analysis of consumer welfare that can take into account the purchase of low- or negative-utility products. Section 5.1 explores how the model compares to other models of stochastic choice that derive from non-expected-utility-maximizing behaviour.

2 Model

There is a set of possible products $N \equiv \{1, \ldots, n\}$. The consumer obtains value $v_k \in (0, \infty)$ from product k. The products are ordered in terms of their value to the consumer, so that $v_1 \geq v_2 \geq \cdots \geq v_n > 0$. The consumer can choose from a subset $M \subseteq N$ of these products. In addition, there is a default 'no-purchase' option with the value $v_0 = 0$ to the consumer. The consumer can always choose this option at a zero price, so the choice set they face is $M \cup \{0\}$. A consumer who buys product k at price p_k obtains utility $u_k = v_k - p_k$.

The consumer can always return a product to the seller after purchase. I assume that there is a common cost of doing so $c \geq 0$. This means that the price cannot exceed the consumer's value for the product plus the return cost: $u_k = v_k - p_k \geq -c$ for all $k \in M$. A firm offering a set of products $M \subseteq N$ sets prices $(p_k)_{k \in M}$ and therefore utilities $(u_k)_{k \in M}$. I assume prices cannot be negative, hence $u_k \leq v_k$ for all $k \in M$. This means that any utility vector induced by prices must lie in a compact set $u_M \in \prod_{k \in M} [-c, v_k]$.

Consumer choices take place in the following dynamic setting. There are discrete time periods $t = 0, 1, 2, \ldots$. In $t = 0$ prices are set. In all subsequent periods a consumer makes a choice from the set $M \cup \{0\}$. The consumer's problem is not intrinsically dynamic. Their current and past choices do not affect their future utility or the future availability of products. Under this assumption, in each period a utility-maximizing consumer chooses an alternative k^* such that

$$k^* \in \underset{k \in M \cup \{0\}}{\arg\max}\ u_k \tag{2}$$

This is independent of the time period t. In contrast, *aspirational consumers* make choices using the following history-dependent procedure. For each alternative $k \in M \cup \{0\}$, the consumer maintains an index tracking utility from that alternative up to time t relative to an *aspiration level*. Denote by $F(k, t)$ the

number of times an alternative k has been chosen up to time period t and by $\alpha_k(t) = \frac{F(k,t)}{t}$ the fraction of times that alternative has been chosen. The vector of choice frequencies up to time t is then denoted by $\alpha(t) = (\alpha_k(t))_{k \in M \cup \{0\}}$. The aspirational consumers' index for product k at time t is

$$U(k,t) = F(k,t)[u_k - \overline{u}(\alpha(t))] \tag{3}$$

where $\overline{u}(\alpha(t))$ is the aspiration level. The aspiration level consists of the average utility received from the set of products over time plus an *aspiration premium* $\gamma > 0$. This premium reflects the consumers' demand that a consumption experience must exceed the average consumption experience they have become used to before being recorded positively in the index. Formally, the aspiration level at time t is

$$\overline{u}(\alpha(t)) = \sum_{k \in M \cup \{0\}} \alpha_k(t) u_k + \gamma \tag{4}$$

In each time period t the consumer chooses an alternative $k(t)$ that maximizes the index

$$k(t) \in \underset{k \in M \cup \{0\}}{\arg\max}\ U(k,t) \tag{5}$$

We have that

$$F(i,t+1) = \begin{cases} F(i,t) + 1 \text{ if } k(t) = i \\ F(i,t) \text{ if } k(t) \neq i \end{cases} \tag{6}$$

This in turn defines an index for the next period: $U(k,t+1)$ for each alternative $k \in M \cup \{0\}$.

2.1 Long-Run Behaviour of Aspirational Consumers

With aspirational consumers, we can show that for any product set M of size $|M| = m$, fixed prices $(p_k)_{k \in M}$, and corresponding utilities $(u_k)_{k \in M}$ the long-run frequency with which each product is consumed converges. Moreover, we can show that the relative long-run frequency with which any alternative is chosen satisfies a balancing condition. Denote the vector of choice frequencies at time t by $\alpha(t) = (\alpha_k(t))_{k \in M \cup \{0\}} \in \Delta^m$ and the long-run choice frequencies by $\alpha^* \equiv (\alpha_k^*)_{k \in M \cup \{0\}} \in \Delta^m$. The counts for each alternative are initialized at some finite initial history $t_0 \geq 1$ such that $\sum_{k \in M \cup \{0\}} F(k,t_0) = t_0$.

Theorem 1. *Fix $\gamma > 0$ and product utilities $u_M \in \prod_{k\in M}[-c, v_k]$. Then for any finite initial history the aspiration-based choice process converges: $\alpha(t) \to \alpha^*$ as $t \to \infty$.*

The long-run choice frequencies $\alpha^* \in \Delta^m$ are such that for any alternative $k \in M \cup \{0\}$

$$\alpha_k^*(\overline{u}(\alpha^*) - u_k) = \frac{\gamma}{m+1} \tag{7}$$

This means for all alternatives $k \in M \cup \{0\}$, $\alpha_k^* > 0$ and $\overline{u}(\alpha^*) > u_k$.

Proof. See Appendix □

In the long run the utility of all alternatives must fall below the aspiration level. If some alternatives had utility greater than or equal to the aspiration level, then one of these alternatives would always be chosen. This would result in the average utility converging to the utility of these high-utility alternatives, and the positive aspiration premium $\gamma > 0$ means the aspiration level would eventually exceed the alternatives' utility.

From the balancing condition (7), we can derive that the relative frequency that any alternative i is chosen compared to j is inversely proportional to the shortfall between the aspiration level and the utility of i.

$$\frac{\alpha_i^*}{\alpha_j^*} = \frac{\overline{u}(\alpha^*) - u_j}{\overline{u}(\alpha^*) - u_i} \tag{8}$$

Thus the product with higher utility is chosen relatively more often. With an aspirational consumer, any long-run choice frequencies α^* can be implemented by the following choice of utilities.

Lemma 1. *Any long-run choice frequencies $\alpha^* \in \operatorname{int}\Delta^m$ for an aspirational consumer can be implemented by choosing product utilities such that for any $i \in M$*

$$u_i(\alpha^*) = \frac{\gamma}{m+1}\frac{1}{1 - \sum_{k\in M}\alpha_k^*} - \frac{\gamma}{(m+1)\alpha_i^*} \tag{9}$$

if $u_M(\alpha^*) = (u_i(\alpha^*))_{i\in M} \in \prod_{i\in M}[-c, v_i]$.

Proof. See Appendix □

We can obtain a converse result: given product set M, any utilities $u_M = (u_k)_{k\in M}$ implement unique long-run choice frequencies α^*. Moreover, for fixed

utilities these choice frequencies converge to putting all weight on the utility-maximizing products as $\gamma \to 0$. For any subset of alternatives $M \subseteq N$ denote the frequency that the product chosen is in $K \subseteq M \cup \{0\}$ by $\alpha^*_{\{K\}} = \sum_{i \in K} \alpha^*_i$.

Proposition 1. *For given product utilities $u_M = (u_k)_{k \in M} \in \prod_{k \in M} [-c, v_k]$ and $\gamma > 0$, an aspirational consumer has a unique long-run aspiration level $\overline{u}^*$ and choice frequencies α^*. The aspiration level and choice frequencies can be written as continuously differentiable functions of u_M and γ.*

$$\overline{u}^*(u_M, \gamma), \quad \alpha^*(u_M, \gamma) = (\alpha^*_i(u_M, \gamma))_{i \in M \cup \{0\}} \tag{10}$$

Define $\hat{M} \equiv \arg\max_{k \in M \cup \{0\}} u_k$. For any subset of products $K \subseteq (M \cup \{0\}) \setminus \hat{M}$

$$\lim_{\gamma \to 0} \alpha^*_{\{K\}}(u_M, \gamma) = 0 \tag{11}$$

Proof. See Appendix □

In general it is difficult to obtain analytical expressions for α^* given utilities of the sort we might use in a Luce model. This is because finding an analytical solution involves solving a polynomial equation of degree $m + 1$. However, when $m = 1$ we can solve for the choice frequencies.

Example 2. There is one product in M, and this product gives utility $u_1 \neq 0$. An aspirational consumer chooses this product with frequency

$$\alpha^*_1 = \frac{1}{2} - \frac{\gamma}{2u_1} + \frac{1}{2u_1}\sqrt{u_1^2 + \gamma^2} \in (0, 1) \tag{12}$$

with the no-purchase option being chosen with remaining frequency $\alpha^*_0 = 1 - \alpha^*_1$. We can solve for these choice frequencies using the quadratic equation that we obtain from Lemma 1.

$$u_1 = \frac{\gamma}{2} \frac{1}{1 - \alpha^*_1} - \frac{\gamma}{2\alpha^*_1} \tag{13}$$

△

2.2 Firm Profits and Consumer Welfare

Given a set of products $M \subseteq N$, a firm chooses prices for each product: p_M. I normalize the firm's cost of production to zero for each good.[2] Given prices p_M inducing long-run choice frequencies $\alpha^*(p_M)$ a firm's long-run average profit is

$$\Pi(\alpha^*(p_M), p_M) = \sum_{k \in M} \alpha_k^*(p_M)\, p_k \tag{14}$$

Write prices in terms of the consumer's value from the product and their utility: $p_k = v_k - u_k$. We can then write the firm's profits in terms of the product utilities u_M.

$$\Pi(\alpha^*(u_M), u_M) = \sum_{k \in M} \alpha_k^*(u_M)(v_k - u_k) \tag{15}$$

For any two vectors of product utilities $u_M \neq u'_M$ that are strictly ranked by long-run average profits, the strict ranking is the same under the overtaking criterion as formalized in Brock (1970).

Similarly, we use as our consumer welfare criterion the long-run average utility.

$$W(\alpha^*|M) \equiv \sum_{i \in M} \alpha_i^* u_i(\alpha^*) \tag{16}$$

where $u_i(\alpha)$ can be calculated from the long-run choice frequencies α using the formula (9) from Lemma 1. This measure of welfare can be easily obtained from the aspiration level by subtracting the aspiration premium: $W(\alpha^*|M) = \overline{u}(\alpha^*) - \gamma$.

3 Monopoly

With rational consumers, the monopolist has a simple optimal pricing strategy. Under this pricing strategy it sells, with frequency one, the highest-value good $k = 1$ to the consumers at price $p_1 = v_1$. This gives the monopolist profit $\Pi = v_1$ and the consumers the utility 0. The monopolist has no reason to restrict the line of products it offers. As long as it offers the highest-margin product, adding or subtracting additional products has no effect on monopoly profits.

With aspirational consumers, the monopolist's profit-maximizing pricing strat-

[2] A model with cost of production $\kappa_k \geq 0$ and consumer value v_k for good k is equivalent to a model where the firm has no cost of production and the consumer derives value $w_k = v_k - \kappa_k$.

egy must implement the following long-run consumer choice frequencies.

Proposition 2. *Fix a product set* $M \subseteq N$ *and let* $\overline{v}_M \equiv \max_{i \in M} v_i$, $i^* \in \arg\max_{i \in M} v_i$, *and define*

$$\overline{u}_L \equiv \frac{\gamma - c}{2} + \sqrt{\frac{1}{4}(\gamma - c)^2 + c\frac{\gamma}{m+1}} \tag{17}$$

We have that $\overline{u}_L \in (0, \infty)$.

Given the product set, the monopolist maximizes its profits by choosing prices to implement a long-run aspiration level $\overline{u}^*$ *that solves the following problem*

$$\Pi^*_{MON}(M) = \max_{\overline{u} \in [\overline{u}_L, \infty)} \hat{\Pi}(\overline{u}|M) \tag{18}$$

where

$$\hat{\Pi}(\overline{u}|M) = \overline{v}_M(1 - \frac{\gamma}{(m+1)\overline{u}}) - \frac{\gamma}{(m+1)(\overline{u}+c)} \sum_{j \in M \setminus \{i^*\}} (\overline{v}_M - v_j) - \overline{u} + \gamma \tag{19}$$

This problem has a unique solution $\overline{u}^* = \arg\max_{\overline{u} \in [\overline{u}_L, \infty)} \hat{\Pi}(\overline{u}|M)$. *At the solution the long-run frequency of purchase for good* i^* *is*

$$\alpha^*_{i^*} = 1 - \frac{\gamma}{(m+1)\overline{u}^*} - \frac{m-1}{m+1}\frac{\gamma}{\overline{u}^* + c} \tag{20}$$

while the frequency of purchase for the remaining goods and the frequency of no purchase are

$$\alpha^*_j = \frac{\gamma}{(m+1)(\overline{u}^* + c)} \text{ for } j \in M \setminus \{i^*\} \text{ and } \alpha^*_0 = \frac{\gamma}{(m+1)\overline{u}^*} \tag{21}$$

Proof. See Appendix □

The monopolist tries to set prices in order to maximize the frequency that consumers choose the product with the highest margin i^*. The frequency of purchase for all other goods is then constrained by the possibility of returning products. The fact that the consumer must obtain at least $-c$ utility from all products means that it is costly for the monopolist to shift frequency share to good i^*, as a lower price for good i^* is required to shift consumers' choice frequency away from the other goods. The relative value it can extract from good i^* versus the other goods, $\sum_{j \in M \setminus \{i^*\}} (\overline{v}_M - v_j)$, is then important in determining whether

the firm sets a price below $\overline{v}_M + c$ for good i^*.

By using our expression for utilities in terms of choice frequencies (9) we can obtain expressions for the monopolist's profit-maximizing utilities and hence prices.

Corollary 1. *For a fixed product set M, under the monopolist's optimal pricing strategy the implemented aspiration level is $\overline{u}^* \in [\overline{u}_L, \infty)$. A profit-maximizing monopolist sets the price for the highest-value product i^* at*

$$p_{i^*} = \overline{v}_M - \overline{u}^* + \frac{\overline{u}^* + c}{1 + \frac{m+1}{\gamma \overline{u}^*}\left((\overline{u}^*)^2 - (\gamma - c)\overline{u}^* - c\frac{\gamma}{m+1}\right)} \leq \overline{v}_M + c \tag{22}$$

and sets prices $p_j = v_j + c$ for all other goods $j \in M \setminus \{i^*\}$. Note that when $\overline{u}^* = \overline{u}_L$ then from (22) we have $p_k = v_k + c$ for all $k \in M$.

Proof. See Appendix □

From Proposition 2, in the absence of a product return cost we can derive a closed-form expression for the long-run welfare of the consumer under the monopolist's profit-maximizing prices.

Corollary 2. *Let $c = 0$. Take a fixed product set M and the corresponding monopoly-profit-maximizing aspiration level $\overline{u}_0^* \in [\overline{u}_L, \infty)$. We have that*

$$\overline{u}_0^* = \max\{\sqrt{\frac{\gamma}{m+1}\left(\overline{v}_M + \sum_{j \in M \setminus \{i^*\}} (\overline{v}_M - v_j)\right)}, \gamma\} \tag{23}$$

The value of the consumer welfare criterion is then $\overline{u}_0^* - \gamma \geq 0$.

Proof. See Appendix □

When $c = 0$, we have that consumer welfare is always weakly greater for aspirational consumers than the zero consumer welfare attained with rational consumers. It can be a strict improvement if the value of products is high relative to the aspiration premium. This is because the aspirational consumer's switching behaviour gives them hold-up power against the monopolist's take-it-or-leave-it offer.

Facing an aspirational consumer means that the product line becomes a relevant margin of design for the monopolist. When the consumers are rational, irrelevant products that do not maximize consumer utility do not matter for firm profits. With aspirational consumers all products are chosen with some frequency.

The following result gives the condition for when the monopolist would not want a new product to be added to the line it is selling. With aspirational consumers, additional products can cannibalize the demand for other products that have higher margins.

Proposition 3. *For any k such that $1 \leq k \leq n$, denote the first k products by $M_k = \{1, \ldots, k\}$. Then for some k we have that M_k is a monopolist's optimal product line. That is, conditional on offering k products the monopolist offers the k products that have the highest values.*

When $c = 0$, adding a product $k+1$ weakly increases the monopolist's profit if and only if

$$v_{k+1} \geq \frac{1}{k+1} \sum_{i=1}^{k} v_i \tag{24}$$

Since $v_1 \geq v_2 \geq \cdots \geq v_n$, once this condition fails it also fails for any subsequent product. Therefore an optimal product line takes the form $M_{k^*} = \{1, \ldots, k^*\}$ where

$$k^* = \max\{k \in N : v_k \geq \frac{1}{k} \sum_{i=1}^{k-1} v_i\} \tag{25}$$

Proof. See Appendix □

The product has to have a sufficiently worse margin than the existing products in order for the monopolist to disprefer adding it. A low-margin product reduces the frequency with which higher-margin products are chosen for little gain. The monopolist benefits from adding duplicates to the product set because they attract consumer demand away from the 'no-purchase' option whilst giving the same margin as the existing products.

4 Competition

In this section, we combine the model of pricing for aspirational consumers with firm competition. Assume each firm sells a single good. Firm k is the firm that produces good $k \in N$, and as before the consumer's value of the goods is ranked $v_1 \geq v_2 \geq \cdots \geq v_n$. As in the monopoly case, we normalize all the firms' costs of production to zero. The firm that produces good $k \in N$ then has the following profit function.

$$\Pi_k(\alpha_k,\, p_k) = \alpha_k\, p_k \tag{26}$$

We contrast firm competition with aspirational consumers with the standard model of Bertrand competition, where firms set prices and the consumer, after observing all prices, chooses the product that gives them the highest utility. This is a simple textbook model which can be adapted from that found in Tirole (1988).[3] This model has Pure-Strategy Nash Equilibria (henceforth PSNE) with a unique outcome. If $v_1 > v_2$ then in PSNE firm 1 sets price $p_1 = v_1 - v_2 > 0$ and serves the entire market, with at least one firm $j \in N \setminus \{1\}$ with $v_j = v_2$ setting $p_j = 0$. Firm 1 obtains profit $v_1 - v_2$ while all other firms obtain zero profit. The consumer gets utility $v_1 - (v_1 - v_2) = v_2$. When $v_1 = v_2$, in PSNE at least two of the firms with value $v_1 = v_2$ set their price equal to zero. All firms obtain zero profit in equilibrium while the consumer gets utility v_1.

With aspirational consumers, a product receives some demand at any price so long as its utility exceeds the return cost: $u_k \geq -c$. Therefore, it is always profitable for a firm to put its product on the market. Thus, it is without loss of generality to consider firm competition without product pruning, so that $M = N$.

We rewrite the firms' profit functions (26) in terms of utilities using $p_k = v_k - u_k$. From Proposition 1 we can write the long-run choice frequencies of an aspirational consumer and the aspiration level as continuously differentiable functions of the vector of utilities u_N. Suppressing the dependence of these functions on γ, denote them $\alpha(u_N)$ and $\overline{u}^*(u_N)$. By condition (7) from Theorem 1 we can write the long-run choice frequency function in terms of $\overline{u}^*(u_N)$. For all $k \in N \cup \{0\}$

$$\alpha_k(u_N) = \frac{\gamma}{n+1} \frac{1}{\overline{u}^*(u_N) - u_k} \tag{27}$$

The profit function of a firm $k \in N$ can then be written entirely in terms of the vector of product utilities $u_N = (u_k, u_{-k})$.

$$\Pi_k(u_k, u_{-k}) = \frac{\gamma}{n+1} \frac{v_k - u_k}{\overline{u}^*(u_k, u_{-k}) - u_k} \tag{28}$$

[3] Specifically, the model in Tirole (1988) Section 5.1, except extended to n firms and with the values now equal to the costs; $v_k = -\kappa_k$, so it becomes a model with heterogeneous goods with no cost of production for firms, rather than a model with homogeneous goods but heterogeneous costs of production among firms as in Tirole (1988). We assume that consumers break utility ties in favour of the higher-value product.

We consider PSNE of a game where each firm chooses the utility of its product (via choosing prices) in order to maximize its profits, given the product utility choices of all other firms. That is, any $u_N^* = (u_1^*, \ldots, u_n^*)$ such that for all $k \in N$

$$u_k^* \in \underset{u_k \in [-c, v_k]}{\arg\max}\ \Pi_k(u_k, u_{-k}^*) \tag{29}$$

For each product $k \in N$, define the long-run frequency share of product k relative to the no-purchase option as a function of utilities u_N.

$$s_k(u_N) = \frac{\alpha_k(u_N)}{\alpha_0(u_N)} \tag{30}$$

This is used in the following result, which characterizes PSNE with aspirational consumers for an arbitrary number of asymmetric firms and general return cost $c \geq 0$.

Proposition 4. *A PSNE of the price-setting game with aspirational consumers exists. A vector of product utilities $u_N^* \in \prod_{k \in N}[-c, v_k]$ is a PSNE if and only if the following conditions hold.*

Define $M_k = \{1, \ldots, k\}$ for $1 \leq k \leq n$ and $M_0 = \varnothing$ for $k = 0$. There is a $0 \leq k^* \leq n$ such that the firms M_{k^*} choose prices such that $u_i^* > -c$ for all $i \in M_{k^*}$, while $u_j^* = -c$ for all $j \in N \setminus M_{k^*}$. The utility choice of any interior firm $i \in M_{k^*}$ satisfies

$$\frac{v_i}{\overline{u}^*(u_N^*)} - 1 = \frac{s_i(u_N^*)}{1 + \sum_{k \in N \setminus \{i\}} s_k(u_N^*)^2} \tag{31}$$

while for any corner firm $j \in N \setminus M_{k^*}$ it holds that

$$\frac{v_j}{\overline{u}^*(u_N^*)} - 1 \leq \frac{s_j(u_N^*)}{1 + \sum_{k \in N \setminus \{j\}} s_k(u_N^*)^2} \tag{32}$$

Proof. See Appendix □

Conditions (31) and (32) reflect the trade-offs each firm faces. The left-hand side is greater when the product's value is high relative to the aspiration level. In this case, the firm has more to gain from increasing the utility (lowering the price) of its product. With a low aspiration level, even a small price reduction can result in a large increase in demand. From the right-hand side, if the firm's share of demand is already high relative to the other alternatives, then a marginal

reduction in price has little impact in increasing it. Together, these conditions imply a lower bound on the aspiration level that is attained under competition.

Using this lower bound, we can build on the characterization result and explore the effect of competition on consumer welfare. We contrast welfare under a monopoly firm that controls the price of all products with decentralized competition where a separate firm controls the price of each product. The first welfare comparison result shows that under a fixed exogenous product line that the monopolist cannot control, competition always leads to weakly higher consumer welfare than under monopoly. This result holds for any non-negative product return cost.

Proposition 5. *Fix any $c \geq 0$. Let α^{com} be the long-run choice frequencies induced by any PSNE of the firm competition game and α^{mon} be the frequencies induced by the monopolist's profit-maximizing pricing. Then*

$$W(\alpha^{com}|N) \geq W(\alpha^{mon}|N) \tag{33}$$

That is, for a fixed product set, consumer welfare is weakly higher under firm competition than under monopoly.

Proof. See Appendix □

This result is non-trivial because with aspirational consumers and a positive return cost a product that gives negative utility can be consumed with positive frequency. These negative-utility products can take demand from positive-utility products through a 'distraction effect' that in principle could reduce consumer welfare if all other product utilities were fixed. However, with a fixed product line, Proposition 5 demonstrates that firms have a greater incentive to offer positive-utility products under competition than under monopoly. Under competition firms do not internalize the costs to other firms of offering higher-utility products, which leads to a higher aspiration level and thus higher consumer welfare.

The proof of Proposition 5 works as follows. By Proposition 2, we have an expression (19) for the maximal profits that the monopolist can attain given a fixed aspiration level and product set. This expression is a strictly concave function of the aspiration level, so there is a unique maximizing aspiration level. Using the conditions (31) and (32) from Proposition 4, we can show that the aspiration level in the firm competition game always exceeds this monopoly profit-maximizing

level. Since the consumer welfare criterion is strictly increasing in the aspiration level, this means consumer welfare is higher under competition than profit-maximizing monopoly.

When the monopolist can endogenously choose which products to offer, a possible benefit to consumers is that the monopolist may remove low- or negative-utility products from the available choice set. This could potentially offset the reduced incentives the monopolist has to offer higher utilities relative to competitive firms. The next result demonstrates that when there are no return costs, competition still results in higher consumer welfare than monopoly even with endogenous product-line selection.

Proposition 6. *Let $c = 0$ and denote the monopolist's profit-maximizing product line by*

$$M^* \in \arg\max_{M \subseteq N} \max_{u_M \in \prod_{k \in M}[0, v_k]} \Pi(\alpha^*(u_M), u_M | M) \tag{34}$$

Let α^{com} be the long-run choice frequencies induced by any PSNE of the firm competition game and α^{mon,M^*} be the long-run choice frequencies induced by the monopolist's profit-maximizing pricing and endogenous product-line selection. Then

$$W(\alpha^{com}|N) \geq W(\alpha^{mon,M^*}|M^*) \tag{35}$$

When $c > 0$ it is possible that there is a PSNE of the firm competition game implementing long-run frequencies $\tilde{\alpha}^{com}$ such that welfare is higher under the monopolist with endogenous product-line selection:

$$W(\tilde{\alpha}^{com}|N) < W(\alpha^{mon,M^*}|M^*) \tag{36}$$

Proof. See Appendix □

We obtain the result with $c = 0$ using the explicit expression (23) for the aspiration level and consumer welfare from Corollary 2. When $c = 0$ the share of demand relative to the outside option must be at least one for any product k: $s_k(u_N) \geq 1$. When combined with the conditions (31) and (32) from the firm competition game, this gives a lower bound on consumer welfare that always exceeds the monopoly level (23).

With positive return costs $c > 0$, a product can give a consumer negative utility. Under competition, a firm will always supply its product to the market because it always receive at least some demand at some price that gives it a positive

margin. A monopolist internalizes the cannibalizing effect that this demand has on other products that the monopolist supplies. The monopolist may therefore gain from removing a product that gives consumers negative utility from the market. This can result in higher consumer welfare.

5 Related Literature

In line with much of the literature in behavioural industrial organization (Spiegler, 2011; Heidhues and Kőszegi, 2018), this paper studies a model where fully rational firms interact with boundedly rational consumers. Gabaix and Laibson (2006), Armstrong and Chen (2009) and Heidhues et al. (2016) study firm competition when some attributes are hidden from myopic consumers. Gamp and Krähmer (2022) study a model with mixed sophisticated and myopic consumers. The presence of myopic consumers is harmful to the sophisticated consumers due to increased search costs, and welfare would be increased by forcing products to meet a minimum quality standard. Spiegler (2006a) and Spiegler (2006b) study firm competition where consumers naively extrapolate from small samples. As in my model, these papers feature consumers who purchase sub-optimal or even negative-utility products in equilibrium. Increased competition can be ineffective at mitigating consumer welfare losses, or may even exacerbate them.

There is some work studying duplicative product proliferation explicitly. An explanation is given by search-slot capture. A seller can ensure that their product is more prominent to consumers who are searching (Armstrong et al., 2009; Armstrong and Zhou, 2011). Huang and Xie (2023) argue this type of model can explain duplicate listings on online food delivery platforms. Brown and Jeon (2021) explain duplication in the Medicare market for prescription drug insurance using the costly information acquisition model of Matějka and McKay (2015). In their model adding clone products can soften the price competition firms face.

The model of boundedly rational consumers I study is adapted from Case-Based Decision Theory (henceforth CBDT), which was introduced and axiomatized by Gilboa and Schmeidler (1995). I adapt a special case of CBDT called Cumulative Utility Consumer Theory explored in Gilboa and Schmeidler (1997) and Gilboa and Pazgal (2001). This paper combines this with an 'aspiration level' which adjusts through time, as in Gilboa and Schmeidler (1996). It is shown in Gilboa and Schmeidler (1996) that if the procedure for adjusting the 'aspiration

level' satisfies certain conditions, then the decision maker asymptotically converges to choosing actions that maximize expected utility. A few papers study variants of CBDT in an IO context. Gans (2002) studies firms that compete in quality levels when faced with CBDT consumers with an exogenous aspiration level. Gilboa and Schmeidler (2001) study how different price paths affect CBDT consumer choices when the aspiration level adjusts endogenously.

While my paper studies multiproduct pricing and product-line selection with boundedly rational consumers, a large literature studies these issues with utility-maximizing consumers. This includes models studying multiproduct firms with vertically differentiated products (Mussa and Rosen, 1978; Champsaur and Rochet, 1989; Johnson and Myatt, 2003, 2006) and models with horizontally differentiated products (Brander and Eaton, 1984; Shaked and Sutton, 1982; Anderson and De Palma, 1992; Klemperer, 1992; Anderson and De Palma, 2006). Two important recent contributions to this literature are Armstrong and Vickers (2018) and Nocke and Schutz (2018). Armstrong and Vickers (2018) studies multiproduct firm pricing by establishing a link with Ramsey pricing. Nocke and Schutz (2018) presents an analysis of multiproduct firm competition that generalizes previous work applying random utility models using an aggregative-game approach. Unlike in their model, the long-run choices of an aspirational consumer cannot be rationalized as a random utility model and do not satisfy the Independence of Irrelevant Alternatives (IIA) property. I discuss this difference from random utility models further in Section 5.1. However, as in Nocke and Schutz (2018) my model of firm competition is an aggregative game, where the aggregator is the aspiration level.

Although distinct from random utility models, the aspirational consumer model is still one where agents make stochastic choices in the long run. In deriving a stochastic choice model from an aspiration-based procedure rather than random utility maximization, I use stochastic approximation tools to characterize long-run choice frequencies. A few papers use these tools when working with similar models of choice. Börgers and Sarin (2000) study an alternative aspiration-based learning model. They show aspiration-based reinforcement learning results in convergence to relative action choice frequencies that are proportional to the payoff from each action, behaviour often called 'probability matching'. Jehiel and Satpathy (2024) consider the dynamics and steady states of choices that are based on using a learning algorithm with coarse valuations. Esponda et al. (2021) and Fudenberg et al.

(2024) analyse long-run choice frequencies when agents have misspecified models and limited stochastic memories, respectively.

5.1 Relationship to Random Utility Models

The long-run choice frequencies of an aspirational consumer can be considered a model of stochastic choice similar to a Luce model. However, there are several important differences, and in general the long-run choice frequencies cannot be rationalized by a random utility model (RUM) as defined by Strzalecki (2025).

The first important difference is that the choice frequencies do not satisfy the Independence of Irrelevant Alternatives (IIA) property of Luce (1959). This is because the aspiration level depends on the choice frequencies. If the aspiration level is fixed exogenously at some level $\overline{u} > \max_{k \in M \cup \{0\}} u_k$, we can write the frequency that any alternative k is chosen as

$$\alpha_k(u_M) = \frac{\frac{1}{\overline{u}-u_k}}{\sum_{j \in M \cup \{0\}} \frac{1}{\overline{u}-u_j}} \tag{37}$$

For a fixed aspiration level, this is the Luce form for choice probabilities. By taking the ratio of the choice frequencies for any two alternatives i, j, it is clear that IIA would be satisfied if the aspiration level were fixed.

$$\frac{\alpha_i(u_M)}{\alpha_j(u_M)} = \frac{\overline{u} - u_j}{\overline{u} - u_i} \tag{38}$$

However, the aspiration level depends endogenously on the choice frequencies through the relationship $\overline{u}(\alpha(u_M)) = \sum_{k \in M \cup \{0\}} \alpha_k(u_M) u_k + \gamma$. This means the ratio (38) depends on the utility of products outside the pair i, j, which violates IIA.

A second important difference is that the long-run choice frequencies of an aspirational consumer violate the regularity assumption required for consistency with a RUM. The choice frequencies satisfy *regularity* if for any two product sets $M_1 \subseteq M_2$, $k \in M_1 \cup \{0\} \subseteq M_2 \cup \{0\}$ implies $\alpha_k(u_{M_2}|M_2) \leq \alpha_k(u_{M_1}|M_1)$, where u_{M_1} is the restriction of u_{M_2}.[4] Block and Marschak (1960) show that choice probabilities that have a random utility representation must satisfy regularity. The

[4] I use the notation $\alpha_k(u_M|M)$ to denote the choice frequency for alternative $k \in M \cup \{0\}$ when the product set is M.

following example demonstrates that an aspirational consumer's long-run choice frequencies do not.

Example 3. Take an aspirational consumer who chooses between espresso, filter coffee and no coffee. We show there are parameters where adding two duplicate instant coffee options to the choice set increases the frequency that espresso is chosen. Formally, the two product sets are $M_1 = \{1, 2\}$ and $M_2 = \{1, 2, 3, 4\}$ where $u_0 = 0$ is no coffee, $u_1 = 640$ is espresso, $u_2 = 525$ is filter and $u_3 = u_4 = 0$ are instant coffee. Let $\gamma = 126$. Table 2 demonstrates the failure of regularity at these parameters.

Table 2: Adding products can cause a regularity violation

	$m_1 = 2,\ \frac{\gamma}{m_1+1} = 42$			$m_2 = 4,\ \frac{\gamma}{m_2+1} = \frac{126}{5}$		
	$\overline{u}(\alpha) - u_k$	α_k	$\alpha_k \cdot [\overline{u}(\alpha) - u_k]$	$\overline{u}(\alpha) - u_k$	α_k	$\alpha_k \cdot [\overline{u}(\alpha) - u_k]$
Espresso (640)	60	7/10	42	35	18/25	126/5
Filter (525)	175	6/25	42	150	21/125	126/5
No Coffee (0)	700	3/50	42	675	14/375	126/5
Instant (0) ×2	—	—	—	675	14/375	126/5
$\overline{u}(\alpha)$	700			675		

Espresso is chosen with greater frequency $\frac{18}{25} > \frac{7}{10}$ in the second choice set. It is easy to verify the choice frequencies and utilities satisfy condition (7) from Theorem 1. By Proposition 1 these are the unique choice frequencies consistent with the product utilities. △

The violations of IIA, regularity and lack of a nesting structure take the choice frequencies generated by aspirational consumers out of the scope of the demand systems considered by Nocke and Schutz (2018). Another important difference with random utility concerns interpretation. A common interpretation of random utility models is that stochastic choice results from random preference shocks, either within a single agent over time or distributed across a population. The apparent randomness in the long-run choices of aspirational consumers instead results from applying a boundedly rational choice procedure.

This paper therefore contributes to the broader literature on stochastic choice without preference shocks. This includes Ke (2018), who characterizes a two-stage model of choice that can distinguish stochastic choice due to mistakes from that due to preference shocks. Aguiar et al. (2016) provide necessary and sufficient

conditions for stochastic choice to be consistent with agents applying a satisficing procedure with random search orders. Kibris et al. (2024) consider a model where stochastic choice results from random reference points where agents have reference-dependent preferences. Like my paper, they study an application to Bertrand competition with differentiated products. Natenzon (2019) presents a model where random choice results from Bayesian inference from context signals. Another paper where stochastic choice results from probabilistic inference is Danenberg and Spiegler (ming). They explore an equilibrium concept where choices are random due to naive inference that ignores sampling error. Dutta (2020) provides an axiomatization of stochastic choice that results from random inability to make certain pairwise comparisons. Finally, Matějka and McKay (2015) present a foundation for multinomial logit choice based on rational inattention. Their model does not exhibit the duplicates property. However, as in the model of aspirational consumers as well as in Natenzon (2019) and Dutta (2020), choice frequencies can violate regularity and hence are not consistent with a random utility representation.

6 Discussion

Policy

Although the model is stylized, the results suggest there are subtleties that are important to account for when analysing policy in markets that have a proliferation of low-quality products. If the set of products available to consumers is fixed, whether by technological, physical or regulatory constraints, then competition benefits consumers in the model. Policies that reduce the costs consumers face when trying to return purchased products to the seller can be complementary with policies that increase competition. In markets with high costs of returning products, policies to increase competition can backfire if they prevent beneficial product pruning by incumbent firms.

Platforms

Many examples of low-quality and duplicative product proliferatfion are found on online marketplace platforms. This paper generally abstracts from explicit modelling of platforms, and the complex coordination and pricing issues that emerge.

An interesting research direction would be to apply the aspirational consumer framework to models of two-sided markets with platform competition such as Rochet and Tirole (2003); Armstrong (2006).

A Appendices

A.1 Proofs

We first prove Lemma 1 and Proposition 1 as they are required for the proof of Theorem 1.

Proof of Lemma 1

Proof. We can rearrange the condition (7) to

$$u_i = \sum_{k \in M} \alpha_k u_k + \gamma(1 - \frac{1}{(m+1)\alpha_i})$$

By multiplying by α_i and summing across $i \in M$, we get

$$\begin{aligned}
&\sum_{k \in M} \alpha_k u_k = (\sum_{k \in M} \alpha_k) \sum_{k \in M} \alpha_k u_k + \gamma((\sum_{k \in M} \alpha_k) - \frac{m}{m+1}) \\
&\Leftrightarrow \\
&\sum_{k \in M} \alpha_k u_k = \frac{\gamma((\sum_{k \in M} \alpha_k) - \frac{m}{m+1})}{1 - \sum_{k \in M} \alpha_k} \\
&\sum_{k \in M} \alpha_k u_k = -\gamma + \frac{\gamma}{m+1} \frac{1}{1 - \sum_{k \in M} \alpha_k}
\end{aligned}$$

Combining this with our expression for u_i gives us

$$u_i = \frac{\gamma}{m+1} \frac{1}{1 - \sum_{k \in M} \alpha_k} - \frac{\gamma}{(m+1)\alpha_i}$$

□

Proof of Proposition 1

Proof. From condition (7), we have that for any $i \in M \cup \{0\}$ frequencies can be written in terms of the aspiration level and utilities.

$$\alpha_i = \frac{\gamma}{(m+1)(\overline{u}(\alpha) - u_i)} \tag{39}$$

Write the aspiration level as $\overline{u}^* = \overline{u}(\alpha)$. We can obtain a polynomial expression in $\overline{u}^*$ by summing over products $i \in M \cup \{0\}$ with expression (39).

$$\sum_{i \in M \cup \{0\}} \alpha_i = \frac{\gamma}{m+1}[\frac{1}{\overline{u}^*} + \sum_{i \in M} \frac{1}{\overline{u}^* - u_i}] = 1 \tag{40}$$

Define the function

$$f(\overline{u}^*, u_M, \gamma) = \frac{1}{\overline{u}^*} + \sum_{i \in M} \frac{1}{\overline{u}^* - u_i} - \frac{m+1}{\gamma} \tag{41}$$

When (40) holds we have that $f(\overline{u}^*, u_M, \gamma) = 0$. Note that since $\alpha_i \in [0,1]$ for all $i \in M \cup \{0\}$, we must have $\overline{u}^* > u_i$. Let the maximum utility be denoted $u^{max} = \max_{i \in M \cup \{0\}} u_i$. The function f is continuously differentiable on $(u^{max}, \infty) \times \prod_{k \in M}[-c, v_k] \times (0, \infty)$.

The function $f(\cdot, u_M, \gamma)$ is decreasing for fixed γ and u_M.

$$f'_{\overline{u}^*}(\overline{u}^*, u_M, \gamma) = -\frac{1}{(\overline{u}^*)^2} - \sum_{i \in M} \frac{1}{(\overline{u}^* - u_i)^2} < 0 \tag{42}$$

Since $\lim_{\overline{u}^* \to u^{max}} f(\overline{u}^*, u_M, \gamma) = \infty$ and $\lim_{\overline{u}^* \to \infty} f(\overline{u}^*, u_M, \gamma) = -\frac{m+1}{\gamma}$, we have that there is a unique $\overline{u}^* \in (u^{max}, \infty)$ such that $f(\overline{u}^*, u_M, \gamma) = 0$. We can thus define a unique function $\overline{u}^*(u_M, \gamma)$ such that $f(\overline{u}^*(u_M, \gamma), u_M, \gamma) = 0$ with $\overline{u}^*(u_M, \gamma) \in (u^{max}, \infty)$.

Since for the function $\overline{u}^*(\cdot)$ we have $f'_{\overline{u}^*}(\overline{u}^*, u_M, \gamma) \neq 0$ we can apply the implicit function theorem.[5] This means the function $\overline{u}^*(.)$ is continuously differentiable for all $\gamma \in (0, \infty)$ and $u_M \in \prod_{k \in M}[-c, v_k]$. Using (39) we can then define a function $\alpha_i(u_M, \gamma)$ for all $i \in M \cup \{0\}$ that is also continuously differentiable for $\gamma \in (0, \infty)$ and $u_M \in \prod_{k \in M}[-c, v_k]$.

For the limit result as $\gamma \to 0$, since $\overline{u}^*(u_M, \gamma) > u^{max}$ for all $\gamma \in (0, \infty)$ we have that it is bounded below. Now consider any product that is not utility-maximizing: $j \in M \cup \{0\}$ such that $u_j < u^{max}$. We can write

$$\alpha_j(u_M, \gamma) = \frac{\gamma}{(m+1)(\overline{u}^*(u_M, \gamma) - u_j)} \tag{43}$$

[5] For more details on the implicit function theorem see Rudin (1976) Theorem 9.28 on page 224.

and since $\overline{u}^*(u_M, \gamma) \geq u^{max} > u_j$ we have

$$\lim_{\gamma \to 0} \alpha_j(u_M, \gamma) = \lim_{\gamma \to 0} \frac{\gamma}{(m+1)(\overline{u}^*(u_M, \gamma) - u_j)} = 0 \tag{44}$$

which gives the result. □

Proof of Theorem 1

Proof. An outline of the proof is as follows. We start with a lemma showing that there is a finite time period after which the consumer must have tried every alternative at least once, and all indices are strictly negative. We then use stochastic approximation techniques to pass the discrete-time process into a continuous-time process that has the same set of accumulation points. The value of maximal index value normalized by time is bounded in an interval, and in continuous time is strictly decreasing until it reaches the lower boundary. Thus we have convergence to the lower boundary where the balancing condition must be satisfied.

Lemma A.1. *There exists a finite time period $\overline{t}$ such that for all $t \geq \overline{t}$ and for every $k \in M \cup \{0\}$ we have $F(k,t) > 0$ and $u_k - \overline{u}(\alpha(t)) < 0$. This means that $U(k,t) < 0$ for all $k \in M \cup \{0\}$ and $t \geq \overline{t}$.*

Proof. We use the following fact throughout.
Fact 1: $\overline{u}(\alpha(t+1)) - \overline{u}(\alpha(t)) = \frac{1}{t+1}[u_{k(t)} - \overline{u}(\alpha(t)) + \gamma]$ where $k(t)$ is the choice the consumer makes at time t. This is because

$$\begin{aligned}
\overline{u}(\alpha(t+1)) - \overline{u}(\alpha(t)) &= \sum_{k \in M} \frac{F(k,t+1)}{t+1} u_k - \sum_{j \in M} \frac{F(j,t)}{t} u_j \\
&= \frac{1}{t+1}[t \sum_{k \in M} \frac{F(k,t+1)}{t} u_k - (t+1) \sum_{j \in M} \frac{F(j,t)}{t} u_j] \\
&= \frac{1}{t+1}[u_{k(t)} + t \sum_{k \in M} \frac{F(k,t)}{t} u_k - (t+1) \sum_{j \in M} \frac{F(j,t)}{t} u_j] \\
&= \frac{1}{t+1}[u_{k(t)} - \sum_{j \in M} \alpha_j(t) u_j] = \frac{1}{t+1}[u_{k(t)} - \overline{u}(\alpha(t)) + \gamma]
\end{aligned}$$

Now suppose for contradiction that there is a set of untried alternatives $\underline{M} \subseteq M \cup \{0\}$ such that $F(j,t) = 0$ for all t if and only if $j \in \underline{M}$. Then the index for any such alternative is such that $U(j,t) = 0$ for all t. This means that the index for the alternative $k(t)$ chosen at any time t must be such that $U(k(t),t) \geq 0$ for all t. Since $k(t) \notin \underline{M}$ we have that $F(k(t),t) > 0$, which means that $u_{k(t)} \geq \overline{u}(\alpha(t))$.

Together with Fact 1 this means that for every t we have $\overline{u}(\alpha(t+1)) - \overline{u}(\alpha(t)) = \frac{1}{t+1}[u_{k(t)} - \overline{u}(\alpha(t)) + \gamma] \geq \frac{\gamma}{t+1}$. Then at the initial history $\alpha(t_0)$, $\overline{u}(\alpha(T)) - \overline{u}(\alpha(t_0)) = \sum_{t=t_0+1}^{T}(\overline{u}(\alpha(t)) - \overline{u}(\alpha(t-1))) \geq \gamma \sum_{t=t_0+1}^{T} \frac{1}{t}$ and $\lim_{T\to\infty} \overline{u}(\alpha(T)) = \infty$ as the harmonic series diverges. However, this contradicts the fact that from the definition of the aspiration level we have $\overline{u}(\alpha(t)) \leq \max_{k\in M\cup\{0\}} u_k + \gamma$ for all t.

Denote the time period at which all alternatives have been tried at least once by t_1. We can use the same argument above to show that there is some time $t^* \geq t_1$ such that all indices are strictly negative: $U(k, t^*) < 0$ for all $k \in M \cup \{0\}$. If this did not hold then $U(k(t), t) \geq 0$ for all $t \geq t_1$, since the chosen product always has a maximal index. The same steps as above then apply.

Finally, we show that if at some time t^* all indices are negative: $U(k, t^*) < 0$ for all $k \in M \cup \{0\}$, then all indices remain negative: $U(k, t) < 0$ for all $k \in M \cup \{0\}$ and $t \geq t^*$. From Fact 1 we have that for any product $j \in M \cup \{0\}$

$$\overline{u}(\alpha(t+1)) - u_j = \overline{u}(\alpha(t)) - u_j + \frac{1}{t+1}[u_{k(t)} - \overline{u}(\alpha(t)) + \gamma]$$

If all indices are negative at time t then $\overline{u}(\alpha(t)) - u_j > 0$. This means if $\gamma \geq \overline{u}(\alpha(t)) - u_{k(t)}$ at time t then $\overline{u}(\alpha(t+1)) - u_j > 0$ also, and we have our result. The remaining case is when $\gamma < \overline{u}(\alpha(t)) - u_{k(t)}$. Since $k(t)$ is the product chosen at time t, it must have maximal index and for all $i \in M \cup \{0\}$

$$\begin{aligned} &F(k(t), t)[u_{k(t)} - \overline{u}(\alpha(t))] \geq F(i, t)[u_i - \overline{u}(\alpha(t))] \\ \Leftrightarrow &F(i, t)[\overline{u}(\alpha(t)) - u_i] \geq F(k(t), t)[\overline{u}(\alpha(t)) - u_{k(t)}] \end{aligned}$$

Since $t \geq F(i, t) > 0$ for all $t \geq t^*$ this implies

$$\overline{u}(\alpha(t)) - u_i \geq \frac{F(k(t), t)}{F(i, t)}(\overline{u}(\alpha(t)) - u_{k(t)}) \geq \frac{1}{t}(\overline{u}(\alpha(t)) - u_{k(t)})$$

which together with Fact 1 gives

$$\begin{aligned} \overline{u}(\alpha(t+1)) - u_i &\geq \frac{1}{t}(\overline{u}(\alpha(t)) - u_{k(t)}) + \frac{1}{t+1}[u_{k(t)} - \overline{u}(\alpha(t)) + \gamma] \\ &= \frac{1}{t(t+1)}[\overline{u}(\alpha(t)) - u_{k(t)} + t\gamma] > 0 \end{aligned}$$

which ensures that all indices are negative in $t+1$ also, and thus are negative in

all future time periods. □

Now define the normalized index for each product and time period $\tilde{U}(k, \alpha(t)) = \frac{1}{t} U(k,t) = \alpha_k(t)[u_k - \overline{u}(\alpha(t))]$ and the maximal index at each time period $V(\alpha(t)) = \max_{k \in M \cup \{0\}} \tilde{U}(k, \alpha(t))$. We have that $\sum_{k \in M \cup \{0\}} \tilde{U}(k, \alpha(t)) = -\gamma$ and hence $V(\alpha(t)) \geq -\frac{\gamma}{m+1}$ for all t.

From Lemma A.1 above, we have that any accumulation point of the process $\alpha(t)$ lies in the subset of the frequency space where the utility of all products lies below the aspiration level

$$A^+ \equiv \{\alpha \in \Delta^m : \overline{u}(\alpha) \geq u_k \text{ for all } k \in M \cup \{0\}\}$$

Combining, this means that for all $\alpha \in A^+$ we have $0 \geq V(\alpha) \geq -\frac{\gamma}{m+1}$. If $V(\alpha) = -\frac{\gamma}{m+1}$ for some $\alpha \in \Delta^m$ then this α satisfies the relative long-run frequency relationship (7) in the Theorem statement.

We now introduce technology from Benaïm et al. (2005) (henceforth BHS) that is used in the proof.[6] For any $\alpha \in \Delta^m$, define the sets $K(\alpha) \equiv \arg\max_{k \in M \cup \{0\}} \tilde{U}(k, \alpha)$ and $H(\alpha) \equiv \text{co}\{e_k : k \in K(\alpha)\}$ where e_k is the unit vector for dimension k and co denotes the convex hull.

Our discrete-time process $\{\alpha(t)\}_{t=1}^{\infty}$ can then be written

$$\alpha(t+1) - \alpha(t) \in \frac{1}{t+1}[H(\alpha(t)) - \alpha(t)]$$

Define the following *differential inclusion*

$$\frac{d\hat{\alpha}}{ds} \in H(\hat{\alpha}) - \hat{\alpha} \equiv G(\hat{\alpha})$$

We have that $G \equiv H(\hat{\alpha}) - \hat{\alpha}$ meets the requirements of Hypothesis 1.1 of BHS, namely that it is a closed set-valued map whose values are nonempty convex subsets of the compact convex set Δ^m. Remark 1.2 of BHS shows how we can extend this differential inclusion so it is defined on the larger domain $\mathbb{R}^{m+1} \supset \Delta^m$. A solution to the differential inclusion is then defined as in BHS.

Definition 1. A solution to the differential inclusion $\frac{d\hat{\alpha}}{ds} \in H(\hat{\alpha}) - \hat{\alpha}$ with initial point $a \in \mathbb{R}^{m+1}$ is an absolutely continuous mapping $\hat{\alpha} : \mathbb{R} \to \mathbb{R}^{m+1}$ such that

[6] See also Benaim et al. (2023), as we use corrected definitions from this note.

$\hat{\alpha}(0) = a$ and

$$\frac{d\hat{\alpha}(s)}{ds} \in H(\hat{\alpha}(s)) - \hat{\alpha}(s)$$

Hypothesis 1.1 is then sufficient for a solution to the differential inclusion to exist. Now define the set of accumulation points of the discrete-time process

$$L \equiv \{\tilde{\alpha} \in \Delta^m : \text{there exists a sequence } t_l \to \infty \text{ such that } \alpha(t_l) \to \tilde{\alpha}\}$$

We adopt the following definition from BHS.

Definition 2. A set $A \subset \mathbb{R}^{m+1}$ is **invariant** for the differential inclusion G if for all $a \in A$ there is a solution $\hat{\alpha}$ to G with $\hat{\alpha}(0) = a$ and such that $\hat{\alpha}(\mathbb{R}) \subset A$.

We can then use their results to show the following.

Lemma A.2. *The set L of accumulation points of the discrete-time process is nonempty, compact, $L \subseteq \mathsf{A}^+$ and invariant for the differential inclusion G.*

Proof. The discrete-time process $\{\alpha(t)\}_{t=1}^{\infty}$ is contained in a compact set Δ^m, hence its set of accumulation points L is compact and nonempty. From Lemma A.1 after some finite time $t_0 \geq 1$ the discrete process $\alpha(t)$ lies in the subset A^+ where the indices for all products are negative, thus $L \subseteq \mathsf{A}^+$.

We can apply the results of BHS as follows. Define the adjusted time scale

$$\tau_{t_0} = 0, \quad \tau_t = \sum_{s=t_0+1}^{t} \frac{1}{s}$$

which we can use to define the affine continuous-time interpolation $w : \mathbb{R}_{\geq 0} \to \mathbb{R}^{m+1}$ where

$$w(\tau_t + s) = \alpha(t) + s\frac{\alpha(t+1) - \alpha(t)}{\tau_{t+1} - \tau_t}, \quad s \in [0, \frac{1}{t+1})$$

From BHS's Proposition 1.3 the interpolation w is a *bounded perturbed solution* of the differential inclusion $\frac{d\hat{\alpha}(s)}{ds} \in H(\hat{\alpha}(s)) - \hat{\alpha}(s)$ (henceforth G). Define the limit set of w

$$L^*(w) = \cap_{T \geq 0} \overline{\{w(s) : s \geq T\}}$$

By BHS Theorem 3.6 the limit set of a bounded perturbed solution such as $L^*(w)$ is *internally chain transitive* for the differential inclusion G. BHS Lemma 3.5 says an internally chain transitive set for G is also *invariant* for G.

To complete the proof we show that $L = L^*(w)$. We have $L \subseteq L^*(w)$ as if $\alpha(t_l) \to \alpha^*$ for some sequence $t_l \to \infty$ then $w(\tau_{t_l}) = \alpha(t_l) \to \alpha^*$ also. For $L \supseteq L^*(w)$, take any sequence $s_l \to \infty$ where $w(s_l) \to \alpha^*$. For each $l \in \mathbb{N}$ we can choose t_l such that $\tau_{t_l} \leq s_l < \tau_{t_l+1}$. We then have

$$||w(s_l) - \alpha(t_l)|| \leq ||\alpha(t_l + 1) - \alpha(t_l)||$$

Given an index-maximizing choice $k(t)$ at period t

$$||w(s_l) - \alpha(t_l)|| \leq ||\alpha(t_l + 1) - \alpha(t_l)|| = \frac{1}{t_l + 1}||e_{k(t_l)} - \alpha(t_l)|| \leq \frac{C}{t_l + 1} \to 0$$

as $s_l \to \infty$

where $C > 0$ is a finite constant and e_i is the unit vector for dimension i. Hence $\alpha(t_l) \to \alpha^*$ as $t_l \to \infty$. □

We now show that the maximal value of the index V is decreasing in time when evaluated along a solution $\hat{\alpha}$ to the differential inclusion G that lies entirely in negative index frequency space A^+. By Lemma A.2 such a solution exists as the set L is invariant for G. At almost every $s \in \mathbb{R}$, we can write the derivative of such a solution $\hat{\alpha}$ as

$$\frac{d\hat{\alpha}(s)}{ds} = h(s) - \hat{\alpha}(s), \quad h(s) \in H(\hat{\alpha}(s))$$

so that h is a probability vector with support on the products that have maximal indices. The normalized index $\tilde{U}(k, \tilde{\alpha}) = \tilde{\alpha}_k[u_k - \overline{u}(\tilde{\alpha})]$ for any product $k \in M \cup \{0\}$ is continuously differentiable on $\tilde{\alpha} \in \Delta^m$. Since a solution $\hat{\alpha}(s)$ is Lipschitz, we then have that $\tilde{U}(k, \hat{\alpha}(s))$ is Lipschitz for all k and as the maximum of a finite set of Lipschitz functions so is $V(\hat{\alpha}(s))$. Thus both $\tilde{U}(k, \hat{\alpha}(s))$ for all k and $V(\hat{\alpha}(s))$ are almost everywhere differentiable in s.

For all products in the index-maximizing set $k \in K(\hat{\alpha}(s))$ for all $s \in \mathbb{R}$ we have that $\tilde{U}(k, \hat{\alpha}(s)) = V(\hat{\alpha}(s))$. This means that when derivatives exist for both

functions we have

$$\frac{d\,\tilde{U}(k,\hat{\alpha}(s))}{ds} = \frac{dV(\hat{\alpha}(s))}{ds} \tag{45}$$

We now derive two equations that are used in the subsequent proof of Lemma A.3. For the first, take the derivative of the aspiration level $\overline{u}(\hat{\alpha}(s)) = \gamma + \sum_{k\in M\cup\{0\}} \hat{\alpha}_k(s)u_k$

$$\frac{d\overline{u}(\hat{\alpha}(s))}{ds} = \sum_{k\in M\cup\{0\}} \frac{d\hat{\alpha}_k(s)}{ds}u_k = \sum_{k\in M\cup\{0\}} h_k(s)u_k - \sum_{k\in M\cup\{0\}} \hat{\alpha}_k(s)u_k$$

Since $\sum_{k\in M\cup\{0\}} \hat{\alpha}_k(s)u_k = \overline{u}(\hat{\alpha}(s)) - \gamma$ we have

$$\frac{d\overline{u}(\hat{\alpha}(s))}{ds} = \gamma + \sum_{k\in M\cup\{0\}} h_k(s)[u_k - \overline{u}(\hat{\alpha}(s))] \tag{46}$$

Similarly, taking the derivative of the normalized index $\tilde{U}(k,\hat{\alpha}(s)) = \hat{\alpha}_k(s)[u_k - \overline{u}(\hat{\alpha}(s))]$ for any product k gives

$$\begin{aligned}\frac{d\,\tilde{U}(k,\hat{\alpha}(s))}{ds} &= \frac{d\hat{\alpha}_k(s)}{ds}[u_k - \overline{u}(\hat{\alpha}(s))] - \hat{\alpha}_k(s)\frac{d\overline{u}(\hat{\alpha}(s))}{ds} \\ &= h_k(s)[u_k - \overline{u}(\hat{\alpha}(s))] - \hat{\alpha}_k(s)[u_k - \overline{u}(\hat{\alpha}(s))] - \hat{\alpha}_k(s)\frac{d\overline{u}(\hat{\alpha}(s))}{ds}\end{aligned}$$

so using the definition of the normalized index then gives equation.

$$\frac{d\,\tilde{U}(k,\hat{\alpha}(s))}{ds} = h_k(s)[u_k - \overline{u}(\hat{\alpha}(s))] - \tilde{U}(k,\hat{\alpha}(s)) - \hat{\alpha}_k(s)\frac{d\overline{u}(\hat{\alpha}(s))}{ds} \tag{47}$$

We then have the following result.

Lemma A.3. *For any solution $\hat{\alpha}$ of the differential inclusion G that is contained in A^+, the function $V(\hat{\alpha}(s))$ is weakly decreasing. Moreover, for any two time periods $s' > s''$ we have that $V(\hat{\alpha}(s'')) > -\frac{\gamma}{m+1}$ implies $V(\hat{\alpha}(s'')) > V(\hat{\alpha}(s'))$.*

Proof. We consider a time s at which both $V(\hat{\alpha}(s))$ and $\tilde{U}(k,\hat{\alpha}(s))$ are differentiable. Since both functions are Lipschitz and hence differentiable almost everywhere, these times have full Lebesgue measure. Denote the frequency of maximizing alternatives at s by

$$\rho(s) = \sum_{k\in K(\hat{\alpha}(s))} \hat{\alpha}_k(s) \tag{48}$$

remembering that $K(\hat{\alpha}(s))$ is the set of index-maximizing alternatives at time s. There are two cases.

Case 1: $0 > V(\hat{\alpha}(s)) \geq -\frac{\gamma}{m+1}$.

For all index-maximizing products $i \in K(\hat{\alpha}(s))$ we have $V(\hat{\alpha}(s)) = \hat{\alpha}_i(s)[u_i - \overline{u}(\hat{\alpha}(s))] < 0$ which means $\hat{\alpha}_i(s) > 0$ hence $\rho(s) > 0$. If $V(\hat{\alpha}(s)) > -\frac{\gamma}{m+1}$, since $\sum_{j \in M \cup \{0\}} \tilde{U}(j, \hat{\alpha}(s)) = -\gamma$ we cannot have that every product is in $K(\hat{\alpha}(s))$ here, otherwise $(m+1)V(\hat{\alpha}(s)) = -\gamma \Rightarrow V(\hat{\alpha}(s)) = -\frac{\gamma}{m+1}$. Thus, for all non-index-maximizing products $j \in (M \cup \{0\}) \setminus K(\hat{\alpha}(s))$ we have that $\tilde{U}(j, \hat{\alpha}(s)) < V(\hat{\alpha}(s)) < 0$ and hence $\hat{\alpha}_j(s) > 0$ also. This means that $\rho(s) < 1$. If $V(\hat{\alpha}(s)) = -\frac{\gamma}{m+1}$, then since $\sum_{j \in M \cup \{0\}} \tilde{U}(j, \hat{\alpha}(s)) = -\gamma$ we must have that all products are index-maximizing, meaning $\rho(s) = 1$.

Combining our derived equation (47) with the fact that $\frac{d\tilde{U}(k,\hat{\alpha}(s))}{ds} = \frac{dV(\hat{\alpha}(s))}{ds}$ for all $k \in K(\hat{\alpha}(s))$ gives

$$\frac{dV(\hat{\alpha}(s))}{ds} + V(\hat{\alpha}(s)) = h_k(s)[u_k - \overline{u}(\hat{\alpha}(s))] - \hat{\alpha}_k(s)\frac{d\overline{u}(\hat{\alpha}(s))}{ds} \tag{49}$$

We can use the fact that $u_k - \overline{u}(\hat{\alpha}(s)) = \frac{V(\hat{\alpha}(s))}{\hat{\alpha}_k(s)}$ for $k \in K(\hat{\alpha}(s))$, then multiply through by $\hat{\alpha}_k(s)$ and sum over all $k \in K(\hat{\alpha}(s))$ to obtain

$$\rho(s)[\frac{dV(\hat{\alpha}(s))}{ds} + V(\hat{\alpha}(s))] = V(\hat{\alpha}(s)) - \Big(\sum_{k \in K(\hat{\alpha}(s))} \hat{\alpha}_k(s)^2\Big)\frac{d\overline{u}(\hat{\alpha}(s))}{ds} \tag{50}$$

If instead we just sum (49) through $k \in K(\hat{\alpha}(s))$ without the other steps we obtain

$$|K(\hat{\alpha}(s))|\Big(\frac{dV(\hat{\alpha}(s))}{ds} + V(\hat{\alpha}(s))\Big) = \sum_{k \in K(\hat{\alpha}(s))} h_k(s)[u_k - \overline{u}(\hat{\alpha}(s))] - \rho(s)\frac{d\overline{u}(\hat{\alpha}(s))}{ds}$$

which we can combine with the first equation we derived above (46) to get

$$|K(\hat{\alpha}(s))|\Big(\frac{dV(\hat{\alpha}(s))}{ds} + V(\hat{\alpha}(s))\Big) = (1 - \rho(s))\frac{d\overline{u}(\hat{\alpha}(s))}{ds} - \gamma \tag{51}$$

We can multiply both sides of this equation (51) by $\sum_{k \in K(\hat{\alpha}(s))} \hat{\alpha}_k(s)^2$, multiply equation (50) by $1 - \rho(s)$ and add the two resulting equations together to eliminate

the $\frac{d\overline{u}(\hat{\alpha}(s))}{ds}$ terms

$$\Big(\frac{dV(\hat{\alpha}(s))}{ds} + V(\hat{\alpha}(s))\Big)\Big(\rho(s)(1-\rho(s)) + |K(\hat{\alpha}(s))| \sum_{k\in K(\hat{\alpha}(s))} \hat{\alpha}_k(s)^2\Big)$$
$$= V(\hat{\alpha}(s))(1-\rho(s)) - \gamma \sum_{k\in K(\hat{\alpha}(s))} \hat{\alpha}_k(s)^2$$

which can be rearranged to obtain

$$\frac{dV(\hat{\alpha}(s))}{ds} = \frac{V(\hat{\alpha}(s))(1-\rho(s))^2 - \Big(\sum_{k\in K(\hat{\alpha}(s))} \hat{\alpha}_k(s)^2\Big)\Big(\gamma + V(\hat{\alpha}(s))|K(\hat{\alpha}(s))|\Big)}{\rho(s)(1-\rho(s)) + |K(\hat{\alpha}(s))| \sum_{k\in K(\hat{\alpha}(s))} \hat{\alpha}_k(s)^2} \tag{52}$$

The denominator is always strictly positive as the set of maximizers is non-empty. If $V(\hat{\alpha}(s)) = -\frac{\gamma}{m+1}$ then we have shown $\rho(s) = 1$ and $|K(\hat{\alpha}(s))| = m+1$ and we have that

$$\frac{dV(\hat{\alpha}(s))}{ds} = \frac{-\Big(\sum_{j\in M\cup\{0\}} \hat{\alpha}_j(s)^2\Big)\Big(\gamma - \gamma\frac{m+1}{m+1}\Big)}{(m+1)\sum_{k\in M\cup\{0\}} \hat{\alpha}_k(s)^2} = 0 \tag{53}$$

If $V(\hat{\alpha}(s)) > -\frac{\gamma}{m+1}$ we have shown that $\rho(s) < 1$. Thus, the first term in the numerator of (52) is negative as $V(\hat{\alpha}(s)) < 0$ by assumption. The second term in the numerator is also negative as

$$-\gamma = \sum_{j\in M\cup\{0\}} \tilde{U}(j, \hat{\alpha}(s)) = V(\hat{\alpha}(s))|K(\hat{\alpha}(s))| + \sum_{j\in (M\cup\{0\})\setminus K(\hat{\alpha}(s))} \tilde{U}(j, \hat{\alpha}(s))$$
$$\Rightarrow \gamma + V(\hat{\alpha}(s))|K(\hat{\alpha}(s))| = -\sum_{j\in (M\cup\{0\})\setminus K(\hat{\alpha}(s))} \tilde{U}(j, \hat{\alpha}(s)) > 0$$

as all non-index-maximizing products must have strictly negative index since $V(\hat{\alpha}(s)) < 0$. Together this means that $\frac{dV(\hat{\alpha}(s))}{ds} < 0$.

Case 2: $V(\hat{\alpha}(s)) = 0$.

Combining the fact that for all $k \in K(\hat{\alpha}(s))$ we have $\frac{d\tilde{U}(k,\hat{\alpha}(s))}{ds} = \frac{dV(\hat{\alpha}(s))}{ds}$ and (47) gives

$$\frac{dV(\hat{\alpha}(s))}{ds} = h_k(s)[u_k - \overline{u}(\hat{\alpha}(s))] - \hat{\alpha}_k(s)\frac{d\overline{u}(\hat{\alpha}(s))}{ds} \tag{54}$$

Taking the sum of this equation over $k \in K(\hat{\alpha}(s))$ and combining with (46) gives

$$\begin{aligned}
&|K(\hat{\alpha}(s))|\frac{dV(\hat{\alpha}(s))}{ds} \\
&= \sum_{k\in K(\hat{\alpha}(s))} h_k(s)[u_k - \overline{u}(\hat{\alpha}(s))] - \rho(s)\Big(\gamma + \sum_{k\in K(\hat{\alpha}(s))} h_k(s)[u_k - \overline{u}(\hat{\alpha}(s))]\Big) \\
&= \big(1-\rho(s)\big) \sum_{k\in K(\hat{\alpha}(s))} h_k(s)[u_k - \overline{u}(\hat{\alpha}(s))] - \rho(s)\gamma \qquad (55)
\end{aligned}$$

which is always weakly below zero as for all $\hat{\alpha}(s) \in \mathsf{A}^+$ we have $u_k - \overline{u}(\hat{\alpha}(s)) \leq 0$ for all $k \in M \cup \{0\}$.

These steps show that $\frac{dV(\hat{\alpha}(s))}{ds} \leq 0$ for almost every time s on every solution in A^+. As $V(\hat{\alpha}(s))$ is absolutely continuous it must therefore be weakly decreasing. We have shown that V is strictly decreasing when $-\frac{\gamma}{m+1} < V(\hat{\alpha}(s)) < 0$. To complete the proof, we show that it is strictly decreasing when starting at some s' such that $V(\hat{\alpha}(s')) = 0$ also.

A solution in which $V(\hat{\alpha}(s)) = 0$ on some interval $s \in (\underline{s}, \overline{s})$ with $\overline{s} > \underline{s}$ means $V(\hat{\alpha}(s)) = 0$ almost everywhere on that interval, and from the equation (55) derived above we must have

$$\rho(s)\gamma = \big(1-\rho(s)\big) \sum_{k\in K(\hat{\alpha}(s))} h_k(s)[u_k - \overline{u}(\hat{\alpha}(s))] \qquad (56)$$

which is impossible unless $\rho(s) = 0$ and $\sum_{k\in K(\hat{\alpha}(s))} h_k(s)[u_k - \overline{u}(\hat{\alpha}(s))] = 0$, since the left-hand side is non-negative and the right-hand side is non-positive. From (46) we then have for almost all $s \in (\underline{s}, \overline{s})$ that

$$\frac{d\overline{u}(\hat{\alpha}(s))}{ds} = \gamma > 0 \qquad (57)$$

meaning $\overline{u}(\hat{\alpha}(s))$ is strictly increasing. Since the support of the probability vector $h(s)$ is contained in $K(\hat{\alpha}(s))$, another implication of $\sum_{k\in K(\hat{\alpha}(s))} h_k(s)[u_k - \overline{u}(\hat{\alpha}(s))] = 0$ is that for all $j \in K(\hat{\alpha}(s))$ it must hold that $u_j = \overline{u}(\hat{\alpha}(s))$. We cannot have that for almost every $s \in (\underline{s}, \overline{s})$ a strictly increasing function $\overline{u}(\hat{\alpha}(s))$ is always equal to one of the finitely many values $\{u_0, \ldots, u_n\}$, so this is a contradiction.

Thus, at any s' where $V(\hat{\alpha}(s')) = 0$ since V is weakly decreasing and we cannot have a non-degenerate interval where it is equal to zero, we must have $V(\hat{\alpha}(s'')) < V(\hat{\alpha}(s'))$ for any $s'' > s'$. □

We can now finish the proof of Theorem 1. Since the set of accumulation points L is compact and the function V is continuous we can define $\alpha^* \in \arg\max_{\alpha \in L} V(\alpha)$. Since L is invariant for G, there is a solution z of G such that $z(0) = \alpha^*$ and $z(\mathbb{R}) \subseteq L$. Thus, for any $T > 0$ we have $z(-T) \in L$ and since α^* is a maximizer $V(z(-T)) \leq V(\alpha^*)$. However, from Lemma A.3 we have that V is decreasing along the solution from $z(-T)$ to $z(0) = \alpha^*$. This means that $V(\alpha^*) \leq V(z(-T))$ and hence $V(\alpha^*) = V(z(-T))$.

Now suppose that $V(\alpha^*) > -\frac{\gamma}{m+1}$ then Lemma A.3 also implies that V is *strictly* decreasing along the solution from $z(-T)$ to $z(0) = \alpha^*$. This means $V(\alpha^*) < V(z(-T))$, a contradiction.

This means that $V(\alpha^*) = -\frac{\gamma}{m+1}$. We have for all $\alpha \in L$ that $V(\alpha) \geq -\frac{\gamma}{m+1}$ since $L \subseteq A^+$. Since α^* maximizes V on L we then have that $V(\alpha) = -\frac{\gamma}{m+1}$ for all $\alpha \in L$.

From the definition of V, this means that $V(\alpha) = \alpha_i[u_i - \overline{u}(\alpha)] = -\frac{\gamma}{m+1}$ for all $i \in M \cup \{0\}$ and $\alpha \in L$. This gives that the expressions across products (7) in the Theorem statement must be satisfied for all $\alpha \in L$. By Proposition 1 we have that these expressions are only satisfied at a unique α^* with $\overline{u}(\alpha^*) > u_i$ and $\alpha_i > 0$ for all $i \in M \cup \{0\}$ by condition (7). This gives our convergence result. □

Proof of Proposition 2

Proof. First note that by Lemma 1, the condition that $u_k \geq -c$ for all $k \in M$ can be written in terms of purchase frequencies

$$\begin{aligned} u_k &= \frac{\gamma}{m+1}\left(\frac{1}{\alpha_0} - \frac{1}{\alpha_k}\right) \geq -c \\ \Rightarrow \alpha_k &\geq \frac{\alpha_0}{1 + \frac{c(m+1)}{\gamma}\alpha_0} \equiv r(\alpha_0) \end{aligned}$$

We then use Lemma 1 to write the profits of the monopolist in terms of purchase frequencies only

$$\Pi(\alpha) = \sum_{k \in M} \alpha_k v_k - \frac{\gamma}{m+1} \frac{1}{1 - \sum_{k \in M} \alpha_k} + \gamma \tag{58}$$

For fixed frequency of no purchase $1 - \sum_{k \in M} \alpha_k$, we can always strictly raise profits by increasing the frequency with which good $i^* \in \arg\max_{j \in M} v_j$ is purchased by $\epsilon > 0$ and reducing the frequency that some good $j \notin \arg\max_{k \in M} v_k$ is

purchased by $\epsilon > 0$. This is true unless $\alpha_j = r(\alpha_0)$ for all $j \notin \arg\max_{k \in M} v_k$. Thus we can only have $\alpha_i > r(\alpha_0)$ for i such that $v_i = \overline{v}_M$, and at a profit-maximizing pricing scheme it is without loss to choose some $i^* \in \arg\max_{j \in M} v_j$ and to set $\alpha_j = r(\alpha_0)$ for all $j \in M \setminus \{i^*\}$.

We can then write an expression for α_{i^*} in terms of α_0

$$\alpha_{i^*} = 1 - \alpha_0 - \sum_{j \in M \setminus \{i^*\}} \alpha_j = 1 - \alpha_0 - (m-1)\frac{\alpha_0}{1 + \frac{c(m+1)}{\gamma}\alpha_0} \tag{59}$$

The satisfaction of the constraint $\alpha_{i^*} \geq r(\alpha_0)$ requires

$$\begin{aligned} \alpha_{i^*} &= 1 - \alpha_0 - (m-1)\frac{\alpha_0}{1 + \frac{c(m+1)}{\gamma}\alpha_0} \geq \frac{\alpha_0}{1 + \frac{c(m+1)}{\gamma}\alpha_0} \\ \Leftrightarrow & 1 - \alpha_0 \geq m\frac{\alpha_0}{1 + \frac{c(m+1)}{\gamma}\alpha_0} \end{aligned} \tag{60}$$

From condition (7) we can write the no-purchase frequency in terms of the aspiration level $\overline{u}$.

$$\alpha_0 = \frac{\gamma}{(m+1)\overline{u}} \tag{61}$$

We can then write the feasibility requirement (60) as a quadratic in the aspiration level.

$$\overline{u}^2 - \overline{u}(\gamma - c) - c\frac{\gamma}{m+1} \geq 0$$

The lowest feasible value for the aspiration level is the positive root $\overline{u}_L$ where this inequality binds. Solving this root gives the expression (17) for $\overline{u}_L$. Thus we have that $\overline{u}_L \leq \overline{u} < \infty$.

Now we rewrite the expression for the monopolist's profits (58) in terms of the no-purchase frequency α_0 using our expressions for α_{i^*} and α_j in terms of α_0 and hence $\overline{u}$ using (61).

$$\hat{\Pi}(\overline{u}|M) = \overline{v}_M\left(1 - \frac{\gamma}{(m+1)\overline{u}}\right) - \frac{\gamma}{(m+1)(\overline{u}+c)} \sum_{j \in M \setminus \{i^*\}} (\overline{v}_M - v_j) - \overline{u} + \gamma$$

Take the first and second derivatives with respect to $\overline{u}$.

$$\frac{d\hat{\Pi}(\overline{u}|M)}{d\overline{u}} = \overline{v}_M \frac{\gamma}{(m+1)\overline{u}^2} + \frac{\gamma}{(m+1)(\overline{u}+c)^2} \sum_{j \in M\setminus\{i^*\}} (\overline{v}_M - v_j) - 1 \tag{62}$$

$$\frac{d^2\hat{\Pi}(\overline{u}|M)}{d\overline{u}^2} = -\overline{v}_M \frac{2\gamma}{(m+1)\overline{u}^3} - \frac{2\gamma}{(m+1)(\overline{u}+c)^3} \sum_{j \in M\setminus\{i^*\}} (\overline{v}_M - v_j) < 0 \tag{63}$$

From the negative second derivative we have that $\hat{\Pi}(\overline{u}|M)$ is strictly concave. The first derivative tends to infinity as $\overline{u} \to 0$ and to –1 as $\overline{u} \to \infty$. Hence it must have an interior root $\overline{u}^{root} \in (0, \infty)$, where $\frac{d\hat{\Pi}(\overline{u}^{root}|M)}{d\overline{u}} = 0$. If $\frac{d\hat{\Pi}(\overline{u}_L|M)}{d\overline{u}} \leq 0$ we have that $\overline{u}^* = \overline{u}_L$ is the maximizing aspiration level in $[\overline{u}_L, \infty)$. Otherwise we have that $\overline{u}^* = \overline{u}^{root} \in [\overline{u}_L, \infty)$ is the unique maximizer from strict concavity. We can obtain the resulting optimal frequency for product i^* from (59) and (61). For all other products $j \in M \setminus \{i^*\}$ we obtain the frequency from $\alpha_j = r(\frac{\gamma}{(m+1)\overline{u}})$.

It remains to verify that at this solution $u_k \leq v_k$ for all $k \in M$. This is clearly true for all $j \in M \setminus \{i^*\}$ as then $u_j = -c$ for those products. It is also clear for the corner solution at which $u_{i^*} = -c$ also. For the interior case, we must have $\frac{d\hat{\Pi}(\overline{u}^*|M)}{d\overline{u}} = 0$ at the solution, which means

$$\begin{aligned}
&1 = \overline{v}_M \frac{\gamma}{(m+1)(\overline{u}^*)^2} + \frac{\gamma}{(m+1)(\overline{u}^*+c)^2} \sum_{j \in M\setminus\{i^*\}} (\overline{v}_M - v_j) \\
&\Rightarrow \overline{u}^* \leq \overline{v}_M \frac{\gamma}{m+1}\Big[\frac{1}{\overline{u}^*} + \frac{(m-1)\overline{u}^*}{(\overline{u}^*+c)^2}\Big] \leq \overline{v}_M \frac{\gamma}{m+1}\Big[\frac{1}{\overline{u}^*} + \frac{m-1}{\overline{u}^*+c}\Big] \\
&\Rightarrow \overline{u}^* \leq \overline{v}_M \Big[\alpha_0 + \sum_{j \in M\setminus\{i^*\}} \alpha_j\Big] \leq \overline{v}_M
\end{aligned}$$

where the last line follows from the condition (7) derivation that $\alpha_0 = \frac{\gamma}{(m+1)\overline{u}^*}$ and $\alpha_j = \frac{\gamma}{(m+1)(\overline{u}^*-u_j)}$ for all $j \in M \setminus \{i^*\}$. Using the expression (9) from Lemma 1 then gives $\overline{v}_M \geq u_{i^*}(\alpha) = \overline{u}^* - \frac{\gamma}{m+1}\frac{1}{\alpha_{i^*}}$. □

Proof of Corollary 1

Proof. This follows from the fact that $p_k = v_k - u_k$ for all $k \in M$, where we combine the expression (9) for utilities in terms of frequencies from Lemma 1 with the long-run profit-maximizing frequencies from Proposition 2. □

Proof of Corollary 2

Proof. When $c = 0$, we have that the lower bound (17) is $\overline{u}_L = \gamma$. We can also take the root of the first derivative of the Proposition 2 expression (19).

$$\frac{d\hat{\Pi}(\overline{u}|M)}{d\overline{u}} = \overline{v}_M \frac{\gamma}{(m+1)\overline{u}^2} + \frac{\gamma}{(m+1)\overline{u}^2} \sum_{j \in M\setminus\{i^*\}} (\overline{v}_M - v_j) - 1 \tag{64}$$

Solving gives the expression in (23) for the case when $\overline{u}_0^* > \gamma$. If $\frac{d\hat{\Pi}(\overline{u}|M)}{d\overline{u}} < 0$ for all $\overline{u} > \overline{u}_L$ then it follows from the first derivative (64) that $\overline{u}_0^* = \gamma \geq \sqrt{\frac{\gamma}{m+1}\big(\overline{v}_M + \sum_{j \in M\setminus\{i^*\}}(\overline{v}_M - v_j)\big)}$. □

Proof of Proposition 3

Proof. For the first part, fix a given product line K of size k and the no-purchase frequency α_0. Denote $\overline{v}_K \equiv \max_{i \in K} v_i$ and $i^* \in \arg\max_{i \in K} v_i$. From Proposition 2, the monopolist's profits can be written

$$\begin{aligned}\hat{\Pi}(\overline{u}|K) &= \overline{v}_K\Big(1 - \frac{\gamma}{(k+1)\overline{u}}\Big) - \frac{\gamma}{(k+1)(\overline{u}+c)} \sum_{j \in K\setminus\{i^*\}} (\overline{v}_K - v_j) - \overline{u} + \gamma \\ &= \overline{v}_K\Big(1 - \frac{\gamma}{(k+1)\overline{u}} - \frac{k-1}{k+1}\frac{\gamma}{\overline{u}+c}\Big) + \frac{\gamma}{(k+1)(\overline{u}+c)} \sum_{j \in K\setminus\{i^*\}} v_j - \overline{u} + \gamma\end{aligned}$$

Moving to a product line $M_k = \{1, \ldots, k\}$ weakly increases profits, as $v_1 \geq \overline{v}_K$ and $\sum_{i=2}^{k} v_i \geq \sum_{j \in K\setminus\{i^*\}} v_j$ while $1 - \frac{\gamma}{(k+1)\overline{u}} - \frac{k-1}{k+1}\frac{\gamma}{\overline{u}+c} \in [0,1]$ if $\overline{u} \in [\overline{u}_L, \infty)$, which holds by Proposition 2.

For the second part, define

$$z_k \equiv \frac{1}{k+1}\Big(v_1 + \sum_{j \in M_k\setminus\{1\}} (v_1 - v_j)\Big).$$

When $c = 0$, we combine (23) from Corollary 2 and (18) from Proposition 2. We can write the monopolist's maximal profit with product line M_k as

$$\Pi^*_{MON}(M_k) = J(z_k),$$

where

$$J(z) = v_1 - z + \big(\max\{\sqrt{z} - \sqrt{\gamma}, 0\}\big)^2.$$

This is a strictly decreasing function, since

$$J'(z) = \begin{cases} -1 & \text{if } z < \gamma \\ -\sqrt{\dfrac{\gamma}{z}} & \text{if } z > \gamma \end{cases}$$

Adding product $k+1$ changes z_k to

$$z_{k+1} = \frac{1}{k+2}\big(v_1 + \sum_{j \in M_k \setminus \{1\}} (v_1 - v_j) + (v_1 - v_{k+1})\big)$$

Therefore, adding product $k+1$ weakly increases monopoly profits if and only if

$$\begin{aligned} & z_{k+1} \leq z_k \\ \iff \quad & \frac{v_1 + \sum_{j \in M_k \setminus \{1\}} (v_1 - v_j) + (v_1 - v_{k+1})}{k+2} \leq \frac{v_1 + \sum_{j \in M_k \setminus \{1\}} (v_1 - v_j)}{k+1}. \end{aligned}$$

Rearranging this gives the inequality condition (24) in the result. □

A.1.1 Facts used in the proofs of Propositions 4, 5 and 6

First, as in the proof of Proposition 1, we have that the aspiration level must satisfy the following equation.

$$f(\overline{u}^*(u_N), u_N, \gamma) = \frac{1}{\overline{u}^*(u_N)} + \sum_{i \in N} \frac{1}{\overline{u}^*(u_N) - u_i} - \frac{n+1}{\gamma} = 0$$

As in that proof, we have $f'_{\overline{u}^*}(\overline{u}^*(u_N), u_N, \gamma) \neq 0$ and so can apply the implicit function theorem. This gives us the partial derivative of the continuously differentiable function $\overline{u}^*(u_N)$ with respect to u_k for any $k \in N$.

$$\begin{aligned} \frac{\partial \overline{u}^*(u_N, \gamma)}{\partial u_k} = -\frac{f'_{u_k}(\overline{u}^*(u_N), u_N, \gamma)}{f'_{\overline{u}^*}(\overline{u}^*(u_N), u_N, \gamma)} &= \frac{\frac{1}{(\overline{u}^*(u_N) - u_k)^2}}{\frac{1}{(\overline{u}^*(u_N))^2} + \sum_{i \in N} \frac{1}{(\overline{u}^*(u_N) - u_i)^2}} \\ &= \frac{\left(\frac{\overline{u}^*(u_N)}{\overline{u}^*(u_N) - u_k}\right)^2}{1 + \sum_{i \in N} \left(\frac{\overline{u}^*(u_N)}{\overline{u}^*(u_N) - u_i}\right)^2} \in (0, 1) \end{aligned} \tag{65}$$

We will also use the fact that we can write the relative share of each product in terms of the aspiration level.

$$s_k(u_N) = \frac{\alpha_k(u_N)}{\alpha_0(u_N)} = \frac{\overline{u}^*(u_N)}{\overline{u}^*(u_N) - u_k} \tag{66}$$

which means that

$$\frac{\partial \overline{u}^*(u_N, \gamma)}{\partial u_k} = \frac{(s_k(u_N))^2}{1 + \sum_{i \in N}(s_i(u_N))^2} \tag{67}$$

and

$$1 + \sum_{i \in N} s_i(u_N) = 1 + \frac{1 - \alpha_0(u_N)}{\alpha_0(u_N)} = \frac{1}{\alpha_0(u_N)} = \frac{(n+1)\overline{u}^*(u_N)}{\gamma} \tag{68}$$

Proof of Proposition 4

Proof. The following step is key to the existence result. We show that the firm's profit function is quasi-concave in the utility of its product and has a unique maximizer.

Lemma A.4. *For each firm $k \in N$, fixing $u_{-k} \in \prod_{j \in N \setminus \{k\}}[-c, v_j]$, the profit function $\Pi_k(., u_{-k})$ is quasi-concave in $u_k \in [-c, v_k]$ and has a unique maximizer $u_k^*(u_{-k}) \in [-c, v_k]$.*

Proof. We can write the profit function of each firm $k \in N$ as follows

$$\Pi_k(u_N) = \frac{\gamma}{n+1}\left[\frac{v_k - \overline{u}^*(u_N)}{\overline{u}^*(u_N) - u_k} + 1\right] \tag{69}$$

then take the partial derivative with respect to u_k

$$\begin{aligned} \frac{\partial \Pi_k(u_N)}{\partial u_k} &= \frac{\gamma}{n+1} \frac{-\frac{\partial \overline{u}^*(u_N)}{\partial u_k}(\overline{u}^*(u_N) - u_k) + (1 - \frac{\partial \overline{u}^*(u_N)}{\partial u_k})(v_k - \overline{u}^*(u_N))}{(\overline{u}^*(u_N) - u_k)^2} \\ &= \frac{\gamma}{n+1} \frac{1 - \frac{\partial \overline{u}^*(u_N)}{\partial u_k}}{(\overline{u}^*(u_N) - u_k)^2}\left[v_k - \overline{u}^*(u_N) - \frac{\frac{\partial \overline{u}^*(u_N)}{\partial u_k}}{1 - \frac{\partial \overline{u}^*(u_N)}{\partial u_k}}(\overline{u}^*(u_N) - u_k)\right] \end{aligned} \tag{70}$$

Using the equation (65) we derived, we have that

$$\frac{\frac{\partial \overline{u}^*(u_N)}{\partial u_k}}{1-\frac{\partial \overline{u}^*(u_N)}{\partial u_k}}(\overline{u}^*(u_N)-u_k) = \frac{\frac{1}{\overline{u}^*(u_N)-u_k}}{\frac{1}{\overline{u}^*(u_N)^2}+\sum_{j\in N\setminus\{k\}}\left(\frac{1}{\overline{u}^*(u_N)-u_j}\right)^2}$$

which can be combined with the partial derivative to get

$$\begin{aligned}&\frac{\partial \Pi_k(u_N)}{\partial u_k}\\ &= \frac{\gamma}{n+1}\frac{1-\frac{\partial \overline{u}^*(u_N)}{\partial u_k}}{(\overline{u}^*(u_N)-u_k)^2}\left[v_k-\overline{u}^*(u_N)-\frac{\frac{1}{\overline{u}^*(u_N)-u_k}}{\frac{1}{\overline{u}^*(u_N)^2}+\sum_{j\in N\setminus\{k\}}\left(\frac{1}{\overline{u}^*(u_N)-u_j}\right)^2}\right]\end{aligned}$$

Since $\frac{\partial \overline{u}^*(u_N)}{\partial u_k} \in (0,1)$, the sign of $\frac{\partial \Pi_k(u_N)}{\partial u_k}$ is equal to the sign of the term in square brackets. This term is strictly decreasing in u_k, since we have shown $\overline{u}^*(u_N)$ is strictly increasing and $\overline{u}^*(u_N)-u_k$ is strictly decreasing in u_k. This means that the sign of $\frac{\partial \Pi_k(u_N)}{\partial u_k}$ can change at most once, and from positive to negative. This means $\Pi_k(u_k, u_{-k})$ is single-peaked in u_k, and hence quasi-concave with a unique maximizer. □

This result and Proposition 1 mean each player has a compact, nonempty and convex strategy set with a continuous and quasi-concave payoff in their own strategy choice. A standard application of the Kakutani-Fan-Glicksberg fixed point theorem then ensures existence.

To obtain the equilibrium characterization, we rewrite our partial derivative equation (70) in terms of relative shares $(s_k(u_N))_{k\in N}$ using the conditions (66) and (67) from subsection A.1.1

$$\begin{aligned}&\frac{\partial \Pi_k(u_N)}{\partial u_k} =\\ &\frac{\gamma}{n+1}\frac{s_k(u_N)^2}{\overline{u}^*(u_N)(1+\sum_{i\in N} s_i(u_N)^2)}\left[\left(\frac{v_k}{\overline{u}^*(u_N)}-1\right)\left(1+\sum_{j\in N\setminus\{k\}} s_j\,(u_N)^2\right)-s_k(u_N)\right]\end{aligned} \tag{71}$$

At any interior optimum for the firm we have that this holds with equality, which gives condition (31). Setting a utility at the upper bound $u_k = v_k$ gives zero profit, while from profit function (28) by setting $u_k = -c$ the firm can always guarantee a positive profit $\frac{\gamma}{n+1}\frac{v_k+c}{\overline{u}^*(-c,u_{-k})+c} > 0$. Hence the only other possibility is the

lower-bound solution $u_k = -c$, which can be optimal if (71) is weakly negative, in which case condition (32) must hold. Since from Lemma A.4 we have that firms have unique maximizers, conditions (31) and (32) are sufficient for a PSNE. This is because if they hold at u_N^* then every firm is best-responding.

We complete the proof by showing necessity. Since (31) and (32) are equivalent to best-response conditions they must hold in any PSNE. It remains to show the cut-off structure holds. If some firm i is interior in equilibrium, and thus chooses utility satisfying (31) by the reasoning above, we show that any firm $j < i$ with higher value $v_j \geq v_i$ must also be interior in equilibrium.

Differentiating (66) gives that

$$\frac{\partial s_k(u_N)}{\partial u_k} = \frac{\frac{\partial \overline{u}^*(u_N)}{\partial u_k}(\overline{u}^*(u_N) - u_k) + (1 - \frac{\partial \overline{u}^*(u_N)}{\partial u_k})\overline{u}^*(u_N)}{(\overline{u}^*(u_N) - u_k)^2} = \frac{\overline{u}^*(u_N) - u_k \frac{\partial \overline{u}^*(u_N)}{\partial u_k}}{(\overline{u}^*(u_N) - u_k)^2}$$

where $\frac{\partial s_k(u_N)}{\partial u_k} > 0$ follows from the facts that $\overline{u}^*(u_N) > u_k$ and $\frac{\partial \overline{u}^*(u_N)}{\partial u_k} \in (0, 1)$ from (65). Hence $s_k(u_N)$ is increasing in u_k. Assume for contradiction that firm j is at the corner $u_j = -c$ while firm i is interior. This means $u_i > u_j$ and hence $s_i(u_N) > s_j(u_N)$. The right-hand side of the interior condition (31) can be written as a function

$$\frac{x}{1 + \sum_{k \in N} s_k(u_N)^2 - x^2}$$

which is strictly increasing. Thus

$$\frac{s_i(u_N)}{1 + \sum_{k \in N \setminus \{i\}} s_k(u_N)^2} > \frac{s_j(u_N)}{1 + \sum_{k \in N \setminus \{j\}} s_k(u_N)^2}$$

Since the right-hand side is larger for firm i than j, firm j on a corner while firm i is interior implies that

$$\frac{v_i}{\overline{u}^*(u_N)} - 1 > \frac{v_j}{\overline{u}^*(u_N)} - 1$$

But this contradicts $v_j \geq v_i$. □

Proof of Proposition 5

Proof. First, note that from Proposition 2 we have an expression (19) for the maximum profit that the monopolist can obtain while implementing aspiration

level $\overline{u}$. As is outlined in Proposition 2 this profit level is attained through a strategy which sets $u_j = -c$ for all $j \in N\backslash\{1\}$ and loads frequency into dimension 1 by setting $u_1 \geq -c$.

Fix a PSNE of the competition game and denote the aspiration level attained in this equilibrium by $\overline{u}^{com}$. We show that at any aspiration level $\overline{u} > \overline{u}^{com}$ the firm's maximum profit is lower than if it chooses prices to implement $\overline{u}^{com}$: $\hat{\Pi}(\overline{u}^{com}|N) > \hat{\Pi}(\overline{u}|N)$. Thus the monopolist will never set an aspiration level above the competitive level. Since our welfare criterion is strictly increasing in the aspiration level, this gives the result.

Take the first and second derivatives of the maximal profit function (19) with respect to $\overline{u}$.

$$\frac{\partial\hat{\Pi}(\overline{u}|N)}{\partial\overline{u}} = v_1\frac{\gamma}{(n+1)\overline{u}^2} + \frac{\gamma}{(n+1)(\overline{u}+c)^2}\sum_{j\in N\backslash\{1\}}(v_1 - v_j) - 1 \tag{72}$$

$$\frac{\partial^2\hat{\Pi}(\overline{u}|N)}{\partial\overline{u}^2} = -v_1\frac{2\gamma}{(n+1)\overline{u}^3} - \frac{2\gamma}{(n+1)(\overline{u}+c)^3}\sum_{j\in N\backslash\{1\}}(v_1 - v_j) < 0 \tag{73}$$

We now show that $\frac{\partial\hat{\Pi}(\overline{u}^{com}|N)}{\partial\overline{u}} \leq 0$, which combined with strict concavity means that $\overline{u}^{com}$ is weakly above the unique maximizer. Since $u_N^* \in \prod_{k\in N}[-c, v_k]$, it induces choice frequencies that are feasible and we have that $\overline{u}^{com} \geq \overline{u}_L$. Hence the monopolist can implement the competitive aspiration level $\overline{u}^{com}$, and reduces its profits by inducing any aspiration level that is greater.

Combining (72) and the derived relationship between the relative shares and aspiration level (68): $1 + \sum_{i\in N} s_i(u_N) = \frac{(n+1)\overline{u}^*(u_N)}{\gamma}$ gives

$$\begin{aligned}
&\frac{\partial\hat{\Pi}(\overline{u}^{com}|N)}{\partial\overline{u}} \leq 0 \\
&\Leftrightarrow v_1\frac{\gamma}{(n+1)(\overline{u}^{com})^2} + \frac{\gamma}{(n+1)(\overline{u}^{com}+c)^2}\sum_{j\in N\backslash\{1\}}(v_1 - v_j) \leq 1 \\
&\Leftrightarrow \frac{v_1}{\overline{u}^{com}}\frac{1}{1+\sum_{i\in N} s_i(u_N)} + \left(\frac{\overline{u}^{com}}{\overline{u}^{com}+c}\right)^2\frac{1}{1+\sum_{i\in N} s_i(u_N)}\sum_{j\in N\backslash\{1\}}\frac{v_1 - v_j}{\overline{u}^{com}} \leq 1 \\
&\Leftrightarrow \frac{v_1}{\overline{u}^{com}} + \left(\frac{\overline{u}^{com}}{\overline{u}^{com}+c}\right)^2\sum_{j\in N\backslash\{1\}}\frac{v_1 - v_j}{\overline{u}^{com}} \leq 1 + \sum_{i\in N} s_i(u_N)
\end{aligned} \tag{74}$$

Since v_1 is the largest value in N, the left-hand side has an upper bound

$$\frac{v_1}{\overline{u}^{com}}\left[1 + \left(\frac{\overline{u}^{com}}{\overline{u}^{com}+c}\right)^2(n-1)\right]$$

From the Proposition 4 conditions for PSNE (31) and (32), we have that for all $i \in N$

$$\frac{v_i}{\overline{u}^{com}} \leq 1 + \frac{s_i(u_N)}{1 + \sum_{k \in N \setminus \{i\}} s_k(u_N)^2}$$

which for $i = 1$ combines with the upper bound to give the following sufficient condition for $\frac{\partial \hat{\Pi}(\overline{u}^{com}|N)}{\partial \overline{u}} \leq 0$.

$$\left[1 + \left(\frac{\overline{u}^{com}}{\overline{u}^{com} + c}\right)^2 (n-1)\right]\left[1 + \frac{s_1(u_N)}{1 + \sum_{k \in N \setminus \{1\}} s_k(u_N)^2}\right] \leq 1 + \sum_{i \in N} s_i(u_N) \quad (75)$$

This can be rewritten as

$$\begin{aligned}
&\frac{s_1(u_N)}{1 + \sum_{k \in N \setminus \{1\}} s_k(u_N)^2} \\
&+ \left(\frac{\overline{u}^{com}}{\overline{u}^{com} + c}\right)^2 (n-1) + \left(\frac{\overline{u}^{com}}{\overline{u}^{com} + c}\right)^2 (n-1)\left[\frac{s_1(u_N)}{1 + \sum_{k \in N \setminus \{1\}} s_k(u_N)^2}\right] \leq \sum_{i \in N} s_i(u_N) \\
&\Leftrightarrow 0 \leq \sum_{i \in N \setminus \{1\}} s_i(u_N) - \left(\frac{\overline{u}^{com}}{\overline{u}^{com} + c}\right)^2 (n-1) \\
&\qquad + s_1(u_N) \frac{\sum_{i \in N \setminus \{1\}} s_i(u_N)^2 - \left(\frac{\overline{u}^{com}}{\overline{u}^{com} + c}\right)^2 (n-1)}{1 + \sum_{i \in N \setminus \{1\}} s_i(u_N)^2} \qquad (76)
\end{aligned}$$

From our expression for the relative product share in terms of the aspiration level (66) and the product return constraint $u_k \geq -c$ we have that

$$s_k(u_N) = \frac{\overline{u}^{com}}{\overline{u}^{com} - u_k} \geq \frac{\overline{u}^{com}}{\overline{u}^{com} + c} \quad (77)$$

which means that the required inequality (76) holds. □

Proof of Proposition 6

Proof. Case when $c = 0$: As before since the consumer welfare criterion is equal to the aspiration level minus γ, if we can show the aspiration level is higher under competition than under monopoly we are done. From Corollary 2, we have that the aspiration level under a monopolist's profit-maximizing pricing with product

set M is

$$\overline{u}(\alpha^{mon,M}) = \max\{\sqrt{\frac{\gamma}{m+1}\big(\overline{v}_M + \sum_{j \in M\setminus\{i^*\}} (\overline{v}_M - v_j)\big)}, \gamma\} \tag{78}$$

Since $\overline{v}_M$ is maximal, this has the following upper bound

$$\overline{u}(\alpha^{mon,M}) \leq \max\{\sqrt{\frac{m}{m+1}\gamma\overline{v}_M}, \gamma\} \tag{79}$$

From Proposition 4, from (31) and (32) under firm competition we have that

$$\frac{v_1}{\overline{u}^*(u_N)} \leq 1 + \frac{s_1(u_N)}{1 + \sum_{k\in N\setminus\{1\}} s_k(u_N)^2} \tag{80}$$

From (66), we have that $s_k(u_N) = \frac{\overline{u}^*(u_N)}{\overline{u}^*(u_N) - u_k} \geq 1$ for all $k \in N$. This means that

$$\begin{aligned} 1 + \frac{s_1(u_N)}{1 + \sum_{k\in N\setminus\{1\}} s_k(u_N)^2} &\leq 1 + \frac{s_1(u_N)}{n} \\ &= \frac{s_1(u_N) + 1 + (n-1)}{n} \\ &\leq \frac{s_1(u_N) + 1 + \sum_{k\in N\setminus\{1\}} s_k(u_N)}{n} \\ &= \frac{1 + \sum_{k\in N} s_k(u_N)}{n} \end{aligned}$$

which combined with (80) means

$$\frac{v_1}{\overline{u}^*(u_N)} \leq \frac{1 + \sum_{k\in N} s_k(u_N)}{n} \tag{81}$$

Earlier we derived the relationship (68) that $1 + \sum_{i\in N} s_i(u_N) = \frac{(n+1)\overline{u}^*(u_N)}{\gamma}$ which together with (81) means

$$\frac{v_1}{\overline{u}^*(u_N)} \leq \frac{n+1}{n}\frac{\overline{u}^*(u_N)}{\gamma} \Rightarrow \overline{u}^*(u_N) \geq \sqrt{\frac{n}{n+1}\gamma v_1}$$

Since $u_k \geq 0$ for all $k \in N$, we also have that $\overline{u}^*(u_N) \geq \gamma$. Hence

$$\overline{u}^*(u_N) \geq \max\{\sqrt{\frac{n}{n+1}\gamma v_1}, \gamma\} \tag{82}$$

As $n \geq m$ and $v_1 \geq \overline{v}_M \equiv \max_{j \in M} v_j$, this lower bound is weakly greater than the upper bound under monopoly (79). Thus we have our result.

Case when $c > 0$: The following example illustrates how when $c > 0$ the consumer welfare criterion can be higher under monopoly with endogenous product-line selection than under competition for some PSNE.

Example 4. Assume there are two products $n = 2$, and that

$$\gamma = 1, \quad c = 1, \quad v_1 = \frac{9}{2}, \quad v_2 = \frac{1}{4}$$

Competition: There is a PSNE where firm 2 is at a corner, firm 1 is interior and the aspiration level is strictly less than $\frac{3}{2}$. If $u_2 = -c = -1$ then we can write the relative shares of each product in terms of the aspiration level. From (66) we have $s_2(u_N) = \frac{\overline{u}}{\overline{u}+1}$ and from (68) we have $s_1(u_N) = 3\overline{u} - s_2(u_N) - 1 = 3\overline{u} - \frac{\overline{u}}{\overline{u}+1} - 1$. From (40) we have that $\frac{\gamma}{3}\frac{1}{\overline{u}} = \frac{1}{3}\frac{1}{\overline{u}} < 1$ and hence $\overline{u} > \frac{1}{3}$.

If $\overline{u} = \frac{3}{2}$, then we have that

$$2 = \frac{v_1}{\overline{u}} - 1 < \frac{s_1(u_N)}{1 + s_2(u_N)^2} = \frac{\frac{29}{10}}{1 + (\frac{3}{5})^2} = \frac{145}{68} \tag{83}$$

and $u_1 = \frac{s_1(u_N)-1}{s_1(u_N)}\overline{u} = \frac{57}{58} > -c = -1$ from rearranging (66). This means the necessary condition (31) for $u_1 = \frac{57}{58} > -c$ in equilibrium does not hold. From Lemma A.4 we have that firm 1's profit functions are single-peaked in u_1, and hence there is a utility $-c \leq u_1^* < \frac{57}{58}$ at which equilibrium condition (31) or (32) holds for firm 1. For firm 2, the equilibrium condition (32) at $u_2 = -1$, for all $\overline{u} \geq \frac{1}{4}$ holds since

$$\frac{\frac{1}{4} - \overline{u}}{\overline{u}} = \frac{v_2}{\overline{u}} - 1 \leq 0 < \frac{s_2(u_N)}{1 + s_1(u_N)^2} \tag{84}$$

Thus both equilibrium conditions are satisfied at $u_1^* \in [-1, \frac{57}{58})$ and $u_2^* = -1$. From (65) we have that the aspiration level $\overline{u}$ is increasing in u_1. Thus we have a PSNE where $\overline{u}^* < \frac{3}{2}$. This means the consumer welfare criterion is strictly less than $\frac{1}{2}$ at this PSNE, since $W(\alpha|N) = \overline{u}(\alpha) - \gamma$.

Monopoly: We use the expression (19), which gives the maximal profits the monopolist can attain for any aspiration level $\overline{u}$. If the monopolist only sells

product 1 then this gives

$$\hat{\Pi}(\overline{u}|\{1\}) = \frac{9}{2}(1-\frac{1}{2\overline{u}}) - \overline{u} + 1 = \frac{27(2\overline{u}-1)(\overline{u}+1)}{12\overline{u}(\overline{u}+1)} - \overline{u} + 1 \tag{85}$$

This is a strictly concave function with a maximum at $\overline{u}^* = \frac{3}{2}$, attained by implementing frequencies $(\alpha_1, \alpha_0) = (1-\frac{\gamma}{(m+1)\overline{u}^*}, \frac{\gamma}{(m+1)\overline{u}^*}) = (\frac{2}{3}, \frac{1}{3})$ through utility $u_1 = \frac{\gamma}{m+1}(\frac{1}{\alpha_0} - \frac{1}{\alpha_1}) = \frac{3}{4}$.

The monopolist's maximal profit from two products given aspiration level $\overline{u}$ is

$$\hat{\Pi}(\overline{u}|\{1,2\}) = \frac{9}{2}(1-\frac{1}{3\overline{u}}) - \frac{1}{3(\overline{u}+1)}\frac{17}{4} - \overline{u} + 1 = \frac{18(3\overline{u}-1)(\overline{u}+1) - 17\overline{u}}{12\overline{u}(\overline{u}+1)} - \overline{u} + 1 \tag{86}$$

Taking the derivative with respect to $\overline{u}$ gives

$$\frac{d\hat{\Pi}(\overline{u}|\{1,2\})}{d\overline{u}} = \frac{3}{2\overline{u}^2} + \frac{1}{3(\overline{u}+1)^2}\frac{17}{4} - 1 \tag{87}$$

which is equal to $\frac{229}{459}$ when $\overline{u} = \frac{9}{8}$ and $-\frac{8}{75}$ when $\overline{u} = \frac{3}{2}$. Since $\hat{\Pi}(\overline{u}|\{1,2\})$ is strictly concave the maximizing aspiration level lies in $(\frac{9}{8}, \frac{3}{2})$. If we take the difference between (85) and (86) for any aspiration level we get

$$\hat{\Pi}(\overline{u}|\{1\}) - \hat{\Pi}(\overline{u}|\{1,2\}) = \frac{8\overline{u}-9}{12\overline{u}(\overline{u}+1)} \tag{88}$$

which is positive for all $\overline{u} \in (\frac{9}{8}, \frac{3}{2})$. For all aspiration levels in this range, the largest implied long-run no-purchase frequency for the single-product monopolist using the relationship $\alpha_0 = \frac{\gamma}{2\overline{u}}$ from condition (7) is $\frac{4}{9}$ which is lower than the feasible upper bound implied by the requirement that $\overline{u} \geq \overline{u}_L$ from (17) (Proposition 2) at these parameters, which is $\alpha_0 = \sqrt{\frac{1}{2}}$. This means that all aspiration levels in the range $(\frac{9}{8}, \frac{3}{2})$ are feasible for the single-product monopolist to implement. Since from (88) above we have that the single-product profits are always larger than the two-product profits at these aspiration levels, we have that the monopolist will only sell product 1. At the monopolist's profit-maximizing price with only product 1 it implements an aspiration level of $\frac{3}{2}$ and hence consumer welfare of $\frac{1}{2}$, which is higher than the consumer welfare achieved under the PSNE with competition outlined above. △

□

References

[1] Aguiar, V. H., M. J. Boccardi, and M. Dean (2016). Satisficing and stochastic choice. *Journal of Economic Theory 166*, 445–482. 21

[2] Anderson, S. P. and A. De Palma (1992). Multiproduct firms: A nested logit approach. *The Journal of Industrial Economics*, 261–276. 19

[3] Anderson, S. P. and A. De Palma (2006). Market performance with multiproduct firms. *The Journal of Industrial Economics 54*(1), 95–124. 19

[4] Armstrong, M. (2006). Competition in two-sided markets. *The RAND journal of economics 37*(3), 668–691. 23

[5] Armstrong, M. and Y. Chen (2009). Inattentive consumers and product quality. *Journal of the European Economic Association 7*(2-3), 411–422. 18

[6] Armstrong, M. and J. Vickers (2018). Multiproduct pricing made simple. *Journal of Political Economy 126*(4), 1444–1471. 19

[7] Armstrong, M., J. Vickers, and J. Zhou (2009). Prominence and consumer search. *The RAND Journal of Economics 40*(2), 209–233. 18

[8] Armstrong, M. and J. Zhou (2011). Paying for prominence. *The Economic Journal 121*(556), F368–F395. 18

[9] Benaïm, M., J. Hofbauer, and S. Sorin (2005). Stochastic approximations and differential inclusions. *SIAM Journal on Control and Optimization 44*(1), 328–348. 28, 29, 30

[10] Benaim, M., J. Hofbauer, and S. Sorin (2023). A note on asymptotic pseudo trajectories for set-valued dynamics. 28

[11] Block, H. D. and J. Marschak (1960). Random orderings and stochastic theories of responses. In I. Olkin, S. G. Ghurye, W. Hoeffding, W. G. Madow, and H. B. Mann (Eds.), *Contributions to Probability and Statistics: Essays in Honor of Harold Hotelling*, pp. 97–132. Stanford, CA: Stanford University Press. 20

[12] Brander, J. A. and J. Eaton (1984). Product line rivalry. *The American Economic Review 74*(3), 323–334. 19

[13] Brock, W. A. (1970). An axiomatic basis for the ramsey-weizsäcker overtaking criterion. *Econometrica: Journal of the Econometric Society*, 927–929. 10

[14] Brown, Z. Y. and J. Jeon (2021). Product proliferation under rational inattention: Application to health insurance. In *AEA Papers and Proceedings*, Volume 111, pp. 554–559. American Economic Association 2014 Broadway, Suite 305, Nashville, TN 37203. 18

[15] Börgers, T. and R. Sarin (2000). Naive reinforcement learning with endogenous aspirations. *International Economic Review 41*(4), 921–950. 19

[16] Champsaur, P. and J.-C. Rochet (1989). Multiproduct duopolists. *Econometrica: Journal of the Econometric Society*, 533–557. 19

[17] Cooper, T. and S. Claxton (2022). Garment failure causes and solutions: Slowing the cycles for circular fashion. *Journal of Cleaner Production 351*, 131394. 2

[18] Creamer, E. Amazon restricts authors from self-publishing more than three books a day after AI concerns. 2

[19] Danenberg, T. and R. Spiegler (forthcoming). A representative-sampling model of stochastic choice. *Journal of Political Economy Microeconomics*. 22

[20] Dutta, R. (2020). Gradual pairwise comparison and stochastic choice. *Theoretical Economics 15*(4), 1335–1364. 22

[21] Esponda, I., D. Pouzo, and Y. Yamamoto (2021). Asymptotic behavior of bayesian learners with misspecified models. *Journal of Economic Theory 195*, 105260. 19

[22] Fudenberg, D., G. Lanzani, and P. Strack (2024). Limited memory, learning, and stochastic choice. *Working paper*. 19

[23] Gabaix, X. and D. Laibson (2006). Shrouded attributes, consumer myopia, and information suppression in competitive markets. *The Quarterly Journal of Economics 121*(2), 505–540. 18

[24] Galak, J. and J. P. Redden (2018). The properties and antecedents of hedonic decline. *Annual review of psychology 69*, 1–25. 4

[25] Galak, J., J. P. Redden, and J. Kruger (2009). Variety amnesia: Recalling past variety can accelerate recovery from satiation. *Journal of Consumer Research 36*(4), 575–584. 4

[26] Gamp, T. and D. Krähmer (2022). Competition in search markets with naive consumers. *The RAND Journal of Economics 53*(2), 356–385. 18

[27] Gans, N. (2002). Customer loyalty and supplier quality competition. *Management Science 48*(2), 207–221. 19

[28] Gilboa, I. and A. Pazgal (2001). Cumulative discrete choice. *Marketing Letters 12*(2), 119–130. 18

[29] Gilboa, I. and D. Schmeidler (1995). Case-based decision theory. *The Quarterly Journal of Economics 110*(3), 605–639. 3, 18

[30] Gilboa, I. and D. Schmeidler (1996). Case-based optimization. *Games and Economic Behavior 15*(1), 1 – 26. 18

[31] Gilboa, I. and D. Schmeidler (1997). Cumulative utility consumer theory. *International Economic Review 38*(4), 737–761. 3, 18

[32] Gilboa, I. and D. Schmeidler (2001). Reaction to price changes and aspiration level adjustments. *Review of Economic Design 6*(2), 215–223. 19

[33] Heidhues, P. and B. Kőszegi (2018). Behavioral industrial organization. *Handbook of Behavioral Economics: Applications and Foundations 1 1*, 517–612. 18

[34] Heidhues, P., B. Kőszegi, and T. Murooka (2016). Inferior products and profitable deception. *The Review of Economic Studies 84*(1), 323–356. 18

[35] Huang, Y. and Y. Xie (2023). Search algorithm, repetitive information, and sales on online platforms. *International Journal of Industrial Organization 88*, 102933. 18

[36] Jehiel, P. and A. Satpathy (2024). Coarse q-learning: Indifference, indeterminacy, and instability. *arXiv preprint arXiv:2412.09321*. 19

[37] Johnson, J. P. and D. P. Myatt (2003). Multiproduct quality competition: Fighting brands and product line pruning. *American Economic Review 93*(3), 748–774. 19

[38] Johnson, J. P. and D. P. Myatt (2006). Multiproduct cournot oligopoly. *The RAND Journal of Economics 37*(3), 583–601. 19

[39] Ke, S. (2018). Rational expectation of mistakes and a measure of error-proneness. *Theoretical Economics 13*(2), 527–552. 21

[40] Kibris, Ö., Y. Masatlioglu, and E. Suleymanov (2024). A random reference model. *American Economic Journal: Microeconomics 16*(1), 155–209. 22

[41] Klemperer, P. (1992). Equilibrium product lines: Competing head-to-head may be less competitive. *The American Economic Review*, 740–755. 19

[42] Luce, R. D. (1959). *Individual Choice Behavior:A Theoretical analysis*. New York, NY, USA: Wiley. 5, 20

[43] Matějka, F. and A. McKay (2015, January). Rational inattention to discrete choices: A new foundation for the multinomial logit model. *American Economic Review 105*(1), 272–98. 18, 22

[44] Mizrachi, M. P. and O. Sharon (2025). Secondhand fashion consumers exhibit fast fashion behaviors despite sustainability narratives. *Scientific Reports 15*(1), 34968. 2

[45] Mussa, M. and S. Rosen (1978). Monopoly and product quality. *Journal of Economic theory 18*(2), 301–317. 19

[46] Natenzon, P. (2019). Random choice and learning. *Journal of Political Economy 127*(1), 419–457. 22

[47] Nocke, V. and N. Schutz (2018). Multiproduct-firm oligopoly: An aggregative games approach. *Econometrica 86*(2), 523–557. 19, 21

[48] Puig, D. (2024). Taste for variety: An intertemporal choice model. *Working paper*. 4

[49] Rana, P. and H. Haddon (2023, march). Uber eats to limit online brand offerings. *Wall Street Journal*. Eastern edition. 2

[50] Ratner, R. K., B. E. Kahn, and D. Kahneman (1999). Choosing less-preferred experiences for the sake of variety. *Journal of consumer research 26*(1), 1–15. 4

[51] Read, D. and G. Loewenstein (1995). Diversification bias: Explaining the discrepancy in variety seeking between combined and separated choices. *Journal of Experimental Psychology: Applied 1*(1), 34. 4

[52] Rochet, J.-C. and J. Tirole (2003). Platform competition in two-sided markets. *Journal of the european economic association 1*(4), 990–1029. 23

[53] Rosenblatt, B. Spotify tightens AI policy and trims catalog. 2

[54] Rudin, W. (1976). *Principles of Mathematical Analysis*. International series in pure and applied mathematics. McGraw-Hill. 25

[55] Shaked, A. and J. Sutton (1982). Relaxing price competition through product differentiation. *The Review of Economic Studies 49*(1), 3–13. 19

[56] Spiegler, R. (2006a). Competition over agents with boundedly rational expectations. *Theoretical Economics 1*(2), 207–231. 18

[57] Spiegler, R. (2006b). The market for quacks. *The Review of Economic Studies 73*(4), 1113–1131. 18

[58] Spiegler, R. (2011). *Bounded rationality and industrial organization*. Oxford University Press. 18

[59] Strzalecki, T. (2025). Stochastic choice theory. *Cambridge Books*. 20

[60] Tirole, J. (1988). *The theory of industrial organization*, Volume 1. MIT press Cambridge, MA. 14